Market Selection with Differential Awareness: The Case of Coarsening*

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Abstract

We analyze a market populated by partially aware agents who perceive a 'coarsening' of the state space and thus disregard the distinction between some states. We first show that the introduction of differential awareness has an impact on prices and allocations as compared to a market with fully aware agents. Moreover, we show that these effects are persistent: in particular, whenever agents have identical correct beliefs and equal discount factors, and the awareness partitions are nested, all agents survive. When agents have heterogeneous beliefs, differential awareness allows agents with wrong beliefs to survive, even when markets are complete and all agents are expected utility maximizers. Provided unawareness is relevant (in a sense we define more precisely), the condition for an agent to survive is that his survival index is at least as large as that of the agents with finer partitions. We also study the impact of an increase in individual awareness and obtain a "paradox of ignorance ": unless the agent can immediately adopt correct beliefs on the newly learned events, increasing his awareness might harm his chances for survival.

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1 Introduction

The question of whether financial markets price assets accurately is of central importance in economics, especially in the light of the rapid increase in the vol-

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ume, value and complexity of financial transactions over recent decades. The strong form of the efficient markets hypothesis (EMH) states that the market price of an asset is the best possible estimate of its value, given all available information, public and private. However, observed behavior of financial markets appears inconsistent with the strong-form EMH in a number of respects. Notable examples are excessive volatility (including apparent 'bubbles' and crashes) and the 'equity premium' and 'risk-free rate' puzzles.

One argument in favor of strong-form EMH, discussed by Blume and Easley (2006) and Sandroni (2000), is the idea that markets favor the best-informed and most rational traders. Trades in a financial market may be seen as 'betting one's beliefs' about the relative probabilities of different states of nature, and the resulting returns on assets. Over time, traders who correctly judge these probabilities and make rational investment choices based on their beliefs will accumulate wealth at the expense of others. In the limit, only these rational well-informed traders will survive, and market prices will reflect their beliefs.

This argument is intuitively appealing, and the central result can be derived under relatively weak conditions[1]. However, the argument raises two major difficulties. First, in simple versions of the model, all but the best-informed traders vanish almost surely (a.s). This appears inconsistent with observed outcomes, where some traders do better than others over the long run, but poorly-informed traders manage to survive. Second, given the infinite multitude of possibilities relevant to market outcomes, the rationality requirements for well-informed traders seem unreasonably strong. *Ad hoc* modifications of rationality assumptions, such as those derived from behavioral observations, appear difficult to specify in a way that sheds light on the problem.

Recent developments in the theory of bounded and differential awareness, (Grant and Quiggin 2013, 2014; Halpern and Rego 2009; Heifetz, Meier and Schipper 2006)[2] provide a way of addressing these problems. The crucial feature of models of bounded awareness is that agents are not aware of all possible states of nature, or of all propositions that might be relevant in formulating probability judgements. Differential awareness arises in market or strategic interactions between such agents, where not all agents have the same awareness. Differential awareness is also relevant in the context of decisions over time, where agents may become aware of new possibilities in the course of the process (Karni and Vierø 2013, 2014).

Grant and Quiggin (2014) distinguish between two forms of bounded awareness: 'coalescence' (more commonly referred to in the literature as 'coarsening')[3], in which some distinctions between states are disregarded, and 'reduc-

[1] This result is conditional on the assumption that markets are complete (Coury and Sciubba 2012), as well as that endowments are bounded (Kogan et al. 2006, 2011; Yan 2008). Its robustness to preference specifications is still being explored (Borovicka 2014; Condie 2008; Da Silva 2011; Easley and Yang 2014; Guerdjikova and Sciubba 2015).

[2] See Schipper (2016) for an extensive bibliography on unawareness.

[3] 'Coalescence' refers to the notion that agents fail to distinguish between different states of nature, effectively coalescing them into a single state. Coarsening refers to the fact that the resulting perceived state space represents a partition of the state space, and that such partitions may be ordered in terms of coarseness or, conversely, refinement. In this paper, we

tion', in which some states of nature are not considered. Both of these possibilities must be distinguished from the case where some agents assign zero probability to events that are objectively possible or that other agents regard as possible. In a companion paper, Guerdjikova and Quiggin (2016) we consider the survival problem in relation to reduced awareness[4]. In this paper, we focus on bounded and differential awareness as coarsening, drawing on the analysis of Heifetz, Meier and Schipper (2006).

We consider the impact of differential awareness, in the sense of coarsening, on allocations and survival in markets. We first construct a simple example of an economy with differential awareness and identical beliefs and show that allocations and prices in general differ from those with fully aware agents. In particular, compared to the standard model where all agents are fully insured against idiosyncratic risk, differential awareness might lead more aware agents to buy only partial insurance against idiosyncratic risk. On the other hand, less aware agents obtain insurance against aggregate risk, even in the absence of differential risk attitudes. In equilibrium, the more aware agent enjoys a higher expected consumption than that of his initial endowment.

We next demonstrate that these effects are persistent in that less aware agents are not driven out of the market. In particular, whenever agents have equal discount factors and identical beliefs, and the awareness partitions are nested, all agents survive. Thus, under these conditions, the coarseness of the partition is irrelevant for survival, even though it alters the equilibrium allocation and even though, ceteris paribus, agents with coarser partitions achieve lower welfare in equilibrium. In the special case, in which awareness is bounded, but not differential, the main results of Blume and Easley (2006) remain valid.

Differential awareness makes a difference when agents have heterogeneous beliefs. Provided unawareness is relevant (in a sense we will define more precisely), an agent whose beliefs are further away from the truth can survive if the agents with beliefs closer to the truth are less aware. The key to the result is that agents may survive either because their probability judgements are more accurate than those of others, or because they can trade on possibilities of which agents with more accurate judgements are unaware. This result is in stark contrast with the results cited above, which preclude belief heterogeneity in bounded economies with complete markets and expected utility maximization.

We next consider the case (arguably the most realistic) where agents' awareness is non-nested, so that no agent is strictly more aware than any other. In particular, we look at an economy, in which each agent is aware of relevant possibilities not considered by any other agent and demonstrate that all agents survive a.s. regardless of their beliefs and discount factors. Adding a fully aware agent with correct beliefs to such an economy implies that the surviving traders, regardless of awareness, must have correct beliefs and equal discount factors.

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will use the term 'coarsening' while maintaining the distinction drawn by Grant and Quiggin (2014).

[4]Modica, Rustichini and Tallon (1998) provide an analysis of a one-period economy with unawareness as reduction.

Finally, we consider two extensions of the model. First, we introduce financial assets and show that the notion of market completeness depends on the awareness structures of the agents. Second, we study the impact of an individual increase in awareness. We show that if all other agents have correct beliefs, adopting correct beliefs on the finer partition upon becoming more aware is a necessary condition for survival. As we explain, however, this condition is far from innocuous. If an agent with correct beliefs is present in the economy, a partially aware agent who becomes aware of new contingencies and who has to use Bayesian updating to learn their correct probabilities will vanish a.s.. We obtain a 'paradox of ignorance': unless the agent can immediately adopt correct beliefs, increasing the agent's awareness will decrease his chances of survival.

2 The Model

2.1 The "True" Model of the Economy

Let $\mathbb{N} = \{0; 1; 2; ..\}$ denote the set of time periods. Uncertainty is modelled through a sequence of random variables $\{\mathcal{S}_t\}_{t\in\mathbb{N}}$ each of which takes values in a finite set S. We set $S_0 = \{s_0\}$, that is, no information is revealed in period 0. Denote by $s_t \in S$ the realization of random variable $\mathcal{S}_t$. Denote by $\Omega = \prod_{t\in\mathbb{N}} S$ the set of all possible observation paths, with representative element $\sigma = (s_0; s_1; s_2 \ldots s_t \ldots)$. Finally denote by $\Omega_t = \prod_{\tau=0}^{t} S$ the collection of all finite paths of length t, with representative element $\sigma_t = (s_0; s_1; s_2 \ldots s_t)$. Each finite observation path σ_t identifies a decision/observation node and the set of all possible observation paths Ω can also be seen as the set of all nodes.

We can represent the information revelation process in this economy through a sequence of finite partitions of the state space Ω. In particular, define the cylinder with base on $\sigma_t \in \Omega_t$, $t \in \mathbb{N}$ as $Z(\sigma_t) = \{\sigma \in \Omega | \sigma = (\sigma_t \ldots)\}$. Let $\mathbb{F}_t = \{Z(\sigma_t) : \sigma_t \in \Omega_t\}$ be a partition of the set Ω. Clearly, $\mathbb{F} = (\mathbb{F}_0 \ldots \mathbb{F}_t \ldots)$ denotes a sequence of finite partitions of Ω such that $\mathbb{F}_0 = \Omega$ and $\mathbb{F}_t$ is finer than $\mathbb{F}_{t-1}$.

Let $\mathcal{F}_t$ be the σ-algebra generated by partition $\mathbb{F}_t$. $\mathcal{F}_0$ is the trivial σ-algebra. Let $\mathcal{F}$ be the σ-algebra generated by $\cup_{t\in\mathbb{N}}\mathcal{F}_t$. It can be shown that $\{\mathcal{F}_t\}_{t\in\mathbb{N}}$ is a filtration.

We define on $(\Omega; \mathcal{F})$ a probability distribution π. Intuitively, π describes the evolution of the state process in the economy. In what follows, for brevity, we abuse notation slightly by denoting $\pi(Z(\sigma_t)) = \pi(\sigma_t) = \pi(s_0; s_1; s_2 \ldots s_t)$. The one-step-ahead probability distribution $\pi(s_{t+1} \mid \sigma_t)$ at node σ_t is determined by:

$$\pi(s_{t+1} \mid \sigma_t) = \pi(s_0 \ldots s_t; s_{t+1} \mid s_0 \ldots s_t) = \frac{\pi(s_0 \ldots s_t; s_{t+1})}{\pi(s_0 \ldots s_t)} \text{ for any } s_{t+1} \in S.$$

In words, $\pi(s_{t+1} \mid \sigma_t)$ is the probability under distribution π^n that the next observation will be s_{t+1}, given that we have reached node σ_t.

We will assume that the true process of the economy is i.i.d. and write $\pi(s_{t+1} = s \mid \sigma_t) =: \pi(s)$. Note that this does not restrict the endowment process to be i.i.d.. The measurability requirements on the endowment process are specified below.

2.2 Modelling Unawareness as Coarsening

In this paper, we think of unawareness as the inability of the agent to form a sufficiently fine perception of the state space. A partially aware agent i will perceive a state space W^i coarser than S, in which some states with potentially different consumption allocations are coalesced into a single perceived state. To understand the process, it is helpful to think in syntactic (propositional terms). Each state in S may be described in terms of the truth values of a set of propositions P describing relevant contingencies, in this case, related to endowments.

An agent may be less aware than another because the set of descriptions available to them is coarser. For example, a relatively unaware agent might consider the proposition 'the economy is (or is not) at full employment', giving rise to a state space with two elements. A more aware agent might distinguish the various phases of the economic cycle, such as 'peak', 'contraction', 'trough' and 'expansion'. An even more aware agent might consider a state space in which the states were indexed by the rate of growth of gross domestic product.

An alternative form of coarsening arises when some agents display 'pure unawareness' of relevant propositions (Li 2008). For example, two agents might have access to the same set of propositions to describe the state of the domestic economy, but only one of them might consider developments in the world economy. The more aware agent would have access to a state space derived as the Cartesian product of the state of the domestic economy and the state of the world economy, while the less aware agent would have access to a coarser quotient space, in which all states of the world economy were treated as indistinguishable. We would expect the less aware agent to display 'home bias' (French and Poterba 1991)

Consider an economy with a finite set I of infinitely lived agents. We now formalize the idea that some agents perceive a coarser state space than the one given by S. In particular, agent i is assumed to be aware of a partition of S given by $W^i = \left\{w_1^i \ldots w_{K_i}^i\right\}$, where each $w_k^i \subseteq S$, $w_k^i \cap w_{k'}^i = \varnothing$ for any $k \neq k'$ and $\cup_{k=1}^{K_i} w_k^i = S$. This is a specific type of unawareness: the agent's perception of the world is coarser than reality in that he cannot distinguish between those states which are grouped in a given w_k^i.

We assume that all fully aware agents have identical information and that the information revelation process for them is represented by the sequence $\mathbb{F}$. A fully aware agent can distinguish any two nodes σ_t and σ_t'. By contrast, a partially aware agent cannot distinguish nodes σ_t and σ_t' if and only if, for every $\tau \leq t$, s_τ, $s_\tau' \in w_{k_\tau}^i$ for some $w_{k_\tau} \in W^i$. Hence, for a partially aware agent, the paths he is aware of can be written as $\Omega^i = \prod_{t \in \mathbb{N}} W^i$ with a representative element $\omega^i = \left(w_0 = \{s_0\}; w_1^i \ldots w_t^i \ldots\right)$. Denote by Ω_t^i the set of paths of length t.

From the point of view of agent i, the information revelation is described by finite partitions of the set Ω^i, $\left(\mathbb{F}_t^i\right)_{t\in\mathbb{N}}$ defined in analogy to $(\mathbb{F}_t)_{t\in\mathbb{N}}$. Note that for each t, $\mathbb{F}_t^i$ is coarser than the corresponding $\mathbb{F}_t$. We will denote by $\mathcal{F}_t^i$ the σ-algebra generated by partition $\mathbb{F}_t^i$. $\mathcal{F}_0^i = \mathcal{F}_0$ is the trivial σ-algebra. Let $\mathcal{F}^i$ be the σ-algebra generated by $\cup_{t\in\mathbb{N}}\mathcal{F}_t^i$. Just as above, $\left\{\mathcal{F}_t^i\right\}_{t\in\mathbb{N}}$ is a filtration.

Agent i's beliefs π^i are defined on $\left(\Omega^i; \mathcal{F}^i\right)$. The one-step ahead probability distribution $\pi^i\left(w_{t+1}^i \mid \omega_t^i\right)$ is defined analogously to $\pi\left(s_{t+1} \mid \sigma_t\right)$.

Obviously, $\mathcal{F}$ is finer than $\mathcal{F}^i$ and hence, the true probability distribution π on $(\Omega; \mathcal{F})$ specifies a probability distribution on $\left(\Omega^i; \mathcal{F}^i\right)$ with

$$\pi\left(\omega_t^i\right) = \pi\left\{\sigma_t \mid s_\tau \in w_\tau^i \text{ for all } \tau \in \{1\ldots t\}\right\} .$$

We will say that i's beliefs are correct if they coincide with the restriction of π to $\left(\Omega^i; \mathcal{F}^i\right)$.

For most of the paper, we will restrict attention to beliefs which describe an i.i.d. process, $\pi^i\left(w_{t+1}^i = w^i \mid \omega_t^i\right) = \pi^i\left(w^i\right)$. We will relax this assumption in Section 5, when we study the impact of an increase in awareness.

There is a single good consumed in positive quantities. We will require that each agent is aware of their consumption stream, which means that the consumption set of i consists of functions $c^i : \Omega \to \prod_{t\in\mathbb{N}}\mathbb{R}_+$ measurable w.r.t. $\left(\Omega^i; \mathcal{F}^i\right)$. In particular, we require the initial endowment of the economy to satisfy this measurability requirement. Each agent i is endowed with a particular $\mathcal{F}_t^i$-measurable consumption plan, called i's endowment stream, and denoted e^i. The total endowment of the economy is denoted by $e = \sum_i e^i$

Agents are assumed to be expected utility maximizers given their knowledge about the economy and their (subjective) beliefs[5]. Agent i's utility function for risk is denoted by u_i and his discount factor is β^i.

We will impose the following assumptions on utility functions and endowments, which are standard in the survival literature:

Assumption 1 All agents are expected utility maximizers with utility functions for risk $u_i : \mathbb{R}_+ \to \mathbb{R}$ which are twice continuously differentiable, strictly concave, and satisfy $\lim_{c\to 0} u_i'(c) = \infty$ and $\lim_{c\to\infty} u_i'(c) = 0$.

Assumption 2 Individual endowments are strictly positive, $e^i\left(\sigma_t\right) > 0$ for all i and σ_t. Aggregate endowments are uniformly bounded away from zero and uniformly bounded from above. Formally, there is an $m > 0$ such that $\sum_{i\in I} e^i(\sigma_t) > m$ for all i, σ_t; moreover, there is an $m' > m > 0$ such that $\sum_{i\in I} e^i(\sigma_t) < m'$ for all σ_t.

Assumption 3 $\pi(s) > 0$ for all $s \in S$ and for all $i \in I$, $\pi^i\left(w^i\right) > 0$ for all $w^i \in W^i$.

[5] An expected utility representation with a coarse subjective state space has been recently axiomatized by Minardi and Savochkin (2016).

Assumption 1 implies that the agent would never choose a 0-consumption in a state he believes to have a positive probability. Assumption 2 ensures that endowments are uniformly bounded away from 0 and above. Given the i.i.d. structure imposed on the true process and on beliefs, Assumption 3 states that one-step-ahead probabilities of all states of the world are positive and that all subjective beliefs assign a positive one-step-ahead probability to every element in their respective partitions. This assumption is analogous to imposing absolute continuity of subjective beliefs w.r.t. the true probability distribution (as in Blume and Easley, 2006). Taken together, Assumptions 1 and 3 ensure that no agent vanishes in finite time.

In economies with bounded endowments and complete markets, and populated by expected utility maximizers, only beliefs and discount factors matter for survival. In particular, if all agents are equally patient, agents with incorrect beliefs vanish a.s. in the presence of agents with correct beliefs. By contrast, in unbounded economies, risk attitudes also matter for survival, and agents with incorrect beliefs can survive. In order to disentangle the effects of unawareness on survival from those of risk attitude, we therefore, restrict our attention to the case of bounded economies.

3 Equilibrium in Financial Markets with Differential Awareness

Our main results are derived on the assumption that agents trade at time 0 over all future contingencies, with no subsequent opportunity for retrading.

Definition 1 *An equilibrium of the economy with differential awareness consists of an integrable price system* $(p(\sigma_t))_{\sigma_t\in\Omega}$ *and a consumption stream* c^i *for every agent* i *such that (i) all agents* $i \in \{1 \dots I\}$ *are maximizing their expected utility given the price system choosing consumption streams measurable relative to their awareness partition and (ii) markets clear:*

$$c^i = \arg\max_{c^i} V_0^i\left(c^i\right) = \arg\max_{c^i} \left\{ \begin{array}{c} u_i\left(c^i\left(\sigma_0\right)\right) + \sum_{t=1}^{\infty} \beta_i^t \sum_{\omega_t^i \in \Omega_t^i} \pi^i\left(\omega_t^i\right) u_i\left(c^i\left(\omega_t^i\right)\right) \\ s.t.\ \sum_{t\in\mathbb{N}} \sum_{\omega_t^i\in\Omega_t^i} \sum_{\sigma_t\in\omega_t^i} p\left(\sigma_t\right) c^i\left(\omega_t^i\right) \\ \leq \sum_{t\in\mathbb{N}} \sum_{\omega_t^i\in\Omega_t^i} \sum_{\sigma_t\in\omega_t^i} p\left(\sigma_t\right) e^i\left(\omega_t^i\right) \end{array} \right\} \tag{1}$$

$$\sum_{i=1}^{I} c^i\left(\sigma_t\right) = \sum_{i=1}^{I} e^i\left(\sigma_t\right) \quad \forall \sigma_t \in \Omega$$

An equilibrium in an economy with (bounded) differential awareness is consistent with the fact that different agents have different perceptions of the state space and, hence, effectively optimize over different sets of commodities (consumption on events ω_t^i, rather than σ_t). The equilibrium can be interpreted in the following way: first, in period 0, before any uncertainty is resolved, all agents choose their consumption paths c^i for all future contingencies of which they are aware. The price of consumption contingent on a coarse contingency ω_t^i is simply the sum of consumption prices over all nodes $\sigma_t \in \omega_t^i$, that is, $\sum_{\sigma_t \in \omega_t^i} p(\sigma_t)$. As time evolves some of the uncertainty is resolved. We assume, however, that agents do not become aware of finer contingencies than those that they perceive given their partition Ω^i. (This assumption is relaxed in Section 5.2). Agents thus do not learn the state σ_t that has occurred at t, but rather the partition that corresponds to the state that has occurred, ω_t^i such that $\sigma_t \in \omega_t^i$.

Proposition 2 *Under Assumptions 1—3, an equilibrium of the economy with coarse contingencies exists. Furthermore, the equilibrium satisfies: for each $i \in I$ and at each ω_t^i, ω_{t+1}^i such that $\pi\left(\omega_{t+1}^i\right) > 0$,*

$$\frac{u_i'\left(c^i\left(\omega_t^i\right)\right)}{\beta_i \pi^i\left(\omega_{t+1}^i \mid \omega_t^i\right) u_i'\left(c^i\left(\omega_{t+1}^i\right)\right)} = \frac{p\left(\omega_t^i\right)}{p\left(\omega_{t+1}^i\right)} =: \frac{\sum_{\sigma_t \in \omega_t^i} p(\sigma_t)}{\sum_{\sigma_{t+1} \in \omega_{t+1}^i} p(\sigma_{t+1})}, \tag{2}$$

where $p(\cdot)$ is the equilibrium price system.

We now provide a simple example to illustrate the impact of coarse contingencies on equilibrium prices and allocations.

Example 3 *Consider an economy with two agents, Ann and Bob. Their initial endowments in each period depend on whether each of them is employed or not. In particular, the economy has 4 states: $S = \{s_1 \ldots s_4\}$. In s_1, A is employed, and B is not, in s_2, B is employed, but not A. In s_3, both agents are unemployed and in s_4, both are employed. Intuitively, states s_1 and s_2 can be interpreted as 'business as usual', in which unemployment is a matter of idiosyncratic risk, whereas in states s_3 and s_4, the economy is subject to aggregate risk (low or high unemployment rates). The initial endowment of an agent is 1 in a state, in which he is unemployed and 2 in a state in which he is employed:*

Initial endowment:	s_1	s_2	s_3	s_4
Ann	2	1	1	2
Bob	1	2	1	2

Assume now that while A is fully aware of S, B is only aware of the partition

$$W^B = \left\{w_1^B = \{s_1; s_3\}; w_2^B = \{s_2; s_4\}\right\}.$$

Hence, while B is conscious of the possibility that he might be unemployed, he does not factor the employment status of A into his reasoning about the economy and is thus ignorant about the aggregate uncertainty the economy faces.
Bob's initial endowment respects the measurability assumption imposed above, that is, $e_{s_1}^B = e_{s_3}^B = 1$ and $e_{s_4}^B = e_{s_4}^B = 2$. This measurability requirement would be violated, if we were to assume that B's partition is given by

$$\tilde{W}^B = \left\{\tilde{w}_1^B = \{s_1; s_2\}; \tilde{w}_2^B = \{s_3; s_4\}\right\},$$

since now $e_{s_1}^B = 1 \neq e_{s_2}^B$ even though $s_1, s_2 \in \tilde{w}_1^B$. Clearly, the partition $\tilde{W}^B$ cannot reflect B's awareness, unless B is ignorant of his own initial endowment.

We assume that the markets in this economy are complete. It follows that each agent can trade on any event he is aware of, and, hence, consumption streams have to satisfy the same measurability requirement as the one imposed on the initial endowments. While markets are a priori complete, this condition will in general restrict the insurance possibilities in the economy and change the equilibrium allocation.

Consider first the case, in which both A and B are fully aware of S. Assuming that both have identical (correct) beliefs π about the realization of the 4 states and strictly concave von-Neumann-Morgenstern utility functions u_A and u_B, the equilibrium of this economy would fully insure both agents against the idiosyncratic risk, that is, $c^A(s_1) = c^A(s_2)$ and $c^B(s_1) = c^B(s_2)$, and hence, $\frac{p_1^*}{\pi(s_1)} = \frac{p_2^*}{\pi(s_2)}$ obtains. As for the allocation across states s_3 and s_4, we know that the less risk-averse agent will partially insure the more risk-averse one against the aggregate risk. If both agents have identical utility functions, no trade across these two states will occur.

Now consider the situation in which B is only partially aware of S and has the partition W^B specified above. The equilibrium allocation described above is no longer feasible, since it specifies $c^B(s_1) > 1 = c^B(s_3)$ and would thus require B to become aware of the distinction between state s_1 and s_3. So what can we say about the equilibrium with unawareness? First, we can show (see the proof of Claim 1 in the Appendix) that neither A, nor B are insured against idiosyncratic risk in equilibrium when B is only partially aware. Second, since u is concave, in equilibrium, $1 < c^B(s_1) < c^B(s_2) < 2$, that is, B buys partial insurance against idiosyncratic risk. This in turn implies that state prices are biased relative to the case of full awareness: $\frac{p_1^*}{\pi(s_1)} < \frac{p_2^*}{\pi(s_2)}$. Finally, if

$$\pi(s_1)\,\pi(s_2) - \pi(s_3)\,\pi(s_4) \le 0 \tag{3}$$

A's expected consumption is higher than her expected initial endowment (see the proof of Claim 2 in the Appendix).

The sufficient condition (3) for A to bear more risk and thus obtain a higher expected consumption than under her initial endowment will hold if all 4 states are equally likely. Alternatively, suppose that the state s_4 has a probability $\pi_4 > \frac{1}{2}$, that is, full employment is the default state of the economy. Assume also that the two states with idiosyncratic risk, s_1 and s_2 are equally probable. $\pi_1 = \pi_2$, that is, the probability that each one of the agents loses their job is the same. In this scenario, condition (3) is satisfied as well.

While the example is formulated as a static one, we may show that,assuming equal discount factors, identical von-Neumann–Morgenstern functions u_A and u_B and an initial endowment i.i.d. over time, the static equilibrium will be replicated in every period t.

We now add (to Ann and Bob) two agents Clara and David. Assume that C has the same initial endowment as A, and that D has the same initial *endowment as B. That is,* $e^C = e^A$, $e^D = e^B$*:*

Initial endowment:	s_1	s_2	s_3	s_4
Clara	2	1	1	2
David	1	2	1	2

however, the awareness partition of Clara is given by

$$W^C = \left\{w_1^C = \{s_1; s_4\}, w_2^C = \{s_2; s_3\}\right\},$$

whereas David is fully aware. Assume that all agents have identical correct beliefs conditional on their awareness partitions and that they are all risk-averse.

Consider first a (sub)economy consisting of only B and C and note that the awareness partitions of B and C preclude any trade between the two of them. From B's point of view the only consumption allocations which he prefers to his initial endowment, and which can be derived through trade, are of the type:

$$\left(c^B(s_1); c^B(s_2); c^B(s_3); c^B(s_4)\right) = (1+a; 2-b; 1+a; 2-b)$$

with $a > 0$, $b > 0$*. However, market clearing implies that the resulting consumption bundle for C specifies*

$$c^B(s_1) = 2 - a \neq c^B(s_4) = 2 + b$$

and is thus inconsistent with her unawareness partition.

In contrast, if A and D were the only agents in the economy, they would fully insure each other across states s_1 *and* s_2 *in equilibrium,* $c^A(s_1) = c^A(s_2)$ *and* $c^D(s_1) = c^D(s_2)$*. This result holds independently of whether their utility functions are identical or not.*[6] *If their utility functions are identical, no trade on the states with aggregate risk,* s_3 *and* s_4 *occurs between them.*

When all four agents are present in the economy, the equilibrium allocation is different. Suppose for simplicity that everyone's beliefs are correct and assign a probability of $\frac{1}{4}$ *to each of the states. First, it is impossible to ensure everybody against idiosyncratic risk in equilibrium, (see the proof of Claim 3 in the Appendix). Second, in general, the presence of partially aware traders in the market (B and C) implies that the fully aware traders A and D cannot be fully insured against idiosyncratic risk, either, (see the proof of Claim 4 in the Appendix).*

Our example demonstrates that markets with partially aware agents exhibit different properties from those populated by fully aware agents. First, in such markets, some of the risk-sharing opportunities cannot be used, due to the measurability requirements on the consumption of partially aware agents. This constraint affects even trades among fully aware agents, who, in the absence of partially aware ones, could have obtained full insurance against idiosyncratic

[6] In the presence of differential bargaining power, which might arise from different risk attitudes, the party with less bargaining power might be required to make a state-independent payment in order to reach agreement on the full insurance bargain.

risk. Second, fully aware agents might provide additional insurance against aggregate risk to partially aware ones, even when both types have identical beliefs and identical risk preferences. Third, in the presence of partially aware agents, state prices might be biased and finally, fully aware agents might obtain higher expected returns than partially aware ones.

4 Survival in Economies with Coarse Contingencies

The insights we gained from the example discussed in the previous section raise the question of whether the impact of partially aware agents on prices and allocations is temporary or permanent. Is it the case that their consumption converges to 0 over time, thus driving the equilibrium allocation to the one that would have obtained had all agents been fully aware? In this section, we will show that partially aware agents can have a long-term impact on prices and risk-sharing.

We define survival as usual:

Definition 4 *Agent i vanishes on a path σ if $\lim_{t\to\infty} c^i(\sigma_t) = 0$. Agent i survives on σ if $\lim_{t\to\infty} \sup c^i(\sigma_t) > 0$.*

In this section, we will assume that Assumptions 1—3 hold, without explicitly stating them in each of the propositions below. We first remark, that in the absence of aggregate risk, partial awareness has no effect on survival:

Remark 5 *In an economy with no aggregate uncertainty, equal discount factors and identical correct beliefs, all agents will be fully insured. Hence, all agents will survive regardless of their level of awareness. In this case, the first-order conditions (2) (with correct beliefs) and the equilibrium allocation coincide with those in a fully aware economy.*

Our first result generalizes the main result of Blume and Easley (2006) to apply to agents with identical awareness partitions.

Proposition 6 *Consider two agents with identical awareness partitions. If the two agents have identical beliefs, but different discount factors, then the agent with the lower discount factor vanishes a.s.. If two agents have identical discount factors and different beliefs, the agent whose beliefs are further away from the truth vanishes a.s.. More generally, the agent with the lower survival index:*

$$\ln \beta_i + \sum_w \pi(w) \ln \frac{\pi(w)}{\pi^i(w)}$$

vanishes a.s..

Our next result concerns agents with nested awareness partitions. It shows that differential awareness alone does not affect survival.

Proposition 7 *Consider a population of agents with nested partitions, equal discount factors and correct beliefs. All agents survive a.s.*

Our result shows that whenever agents have equal discount factors and correct beliefs, and the awareness partitions are nested, the coarseness of the partition is irrelevant for survival. In fact, all agents survive. We can relate this result to Example 3. Recall that in a one-period economy with fully aware Ann and partially aware Bob, insurance against idiosyncratic risk did not obtain in equilibrium. In contrast, the fully aware agent insured the partially aware one against some of the aggregate risk. The result above implies that these features of the economy will persist in the long-run, as long as both agents have equal survival indices.

Even though the coarseness of the agent's partition is irrelevant for survival, it has an impact on the agent's welfare as the next proposition shows:

Proposition 8 *Consider an economy with coarse contingencies and assume that two agents i and j have nested awareness partitions such that the partition of i is finer than that of j, identical endowments $e^i = e^j = e$, identical utility functions u, identical discount factors β and identical beliefs π restricted to Ω^j. In any equilibrium of the economy with equilibrium allocation c and price system p, $V_0^i\left(c^i\right) \geq V_0^j\left(c^j\right)$.*

Ceteris paribus, an agent with a coarser partition will obtain a lower welfare in equilibrium than an agent with a higher level of awareness. Intuitively, the more aware agent will benefit from the larger set of trades that he can engage in and will obtain a higher utility from consumption. Note, however, that the weak inequality cannot be replaced by a strict one. For example, if i and j are the only agents in the population, no trade will occur in equilibrium and their welfare will be identical.

We next examine the impact of heterogeneity in discount factors and beliefs on survival when agents have nested partitions. We first show that an agent with a coarser partition can only survive if his survival index is at least as large as that of the agent with finer partition. In particular, for given identical beliefs, the partially aware agent can survive only if his discount factor is at least as high as that of the fully aware agent and, for given identical discount factors, the partially aware agent can only survive only if his beliefs are at least as close to the truth as those of the fully aware agent.

Proposition 9 *If an agent i who is partially aware has a strictly lower survival index than an agent j who has a finer awareness partition,*

$$\ln\frac{\beta_j}{\beta_i} + \left(\sum_{w^i\in W^i} \pi\left(w^i\right)\ln\frac{\pi\left(w^i\right)}{\pi^i\left(w^i\right)} - \sum_{w^i\in W^i} \pi\left(w^i\right)\ln\frac{\pi\left(w^i\right)}{\pi^j\left(w^i\right)}\right) > 0$$

i vanishes a.s.

This result shows that in order for an agent i with a coarser partition than agent j to survive, it has to be that:

$$\ln \frac{\beta_j}{\beta_i} + \left(\sum_{w^i \in W^i} \pi\left(w^i\right) \ln \frac{\pi\left(w^i\right)}{\pi^i\left(w^i\right)} - \sum_{w^i \in W^i} \pi\left(w^i\right) \ln \frac{\pi\left(w^i\right)}{\pi^j\left(w^i\right)} \right) \leq 0.$$

Hence, we will consider an economy, in which the agents with coarser partitions have (weakly) larger survival indices. To formulate our result, we will have to understand when unawareness matters in the long-term. We will use the following definition:

Definition 10 *The unawareness of agent i, given by the partition Ω^i, is irrelevant in the limit if for any $\omega^i \in \Omega^i$ and any σ, $\sigma' \in \omega^i$*

$$\lim_{t\to\infty} e\left(\sigma_t\right) - e\left(\sigma_t'\right) = 0 \; .$$

The unawareness of agent i, given by the partition Ω^i is relevant in the limit if for some $w^i \in W^i$, s and $s' \in w^i$, there is an $\epsilon > 0$ such that for any σ, $\sigma' \in \omega^i$,

$$\lim_{t\to\infty} \sup\left[e\left(\sigma_t; s\right) - e\left(\sigma_t'; s'\right)\right] > \epsilon \; . \tag{4}$$

The unawareness of agent i is considered irrelevant if in the limit, the total endowment of the economy is measurable w.r.t. agent i's awareness partition. Such an agent is aware of the total endowment process of the economy in the limit. In contrast, agent i's unawareness is relevant even in the limit, if there are at least two states that i cannot distinguish and in which the total endowment of the economy remains distinct.

Proposition 11 *Consider a population of agents with nested partitions Ω^1 strictly finer than Ω^2... strictly finer than Ω^n and ordered survival indices such that: either*

$$\ln \beta_k - \sum_{w^{k'}} \pi\left(w^{k'}\right) \ln \frac{\pi\left(w^{k'}\right)}{\pi^k\left(w^{k'}\right)} < \ln \beta_{k'} - \sum_{w^{k'}} \pi\left(w^{k'}\right) \ln \frac{\pi\left(w^{k'}\right)}{\pi^{k'}\left(w^{k'}\right)}$$

or $\beta_k = \beta_{k'}$ and $\pi^k\left(w^{k'}\right) = \pi^{k'}\left(w^{k'}\right)$ for all $w^{k'} \in W^{k'}$ holds for all $k < k'$ Assume that $\tilde{k}$ is the agent with the finest partition, whose unawareness is relevant in the limit ($\tilde{k} > 1$), whereas the unawareness of all agents $i < \tilde{k}$ is irrelevant in the limit. Then all agents $\left(\tilde{k}-1\right); \tilde{k}..n$ survive a.s. Agent $k < \left(\tilde{k}-1\right)$ survives a.s. if

$$\left\{ \ln \frac{\beta_{\tilde{k}-1}}{\beta_k} - \sum_{w^{\tilde{k}}} \pi\left(w^{\tilde{k}-1}\right) \ln \frac{\pi^k\left(w^{\tilde{k}-1}\right)}{\pi^{\tilde{k}-1}\left(w^{\tilde{k}-1}\right)} \right\} = 0$$

and vanishes a.s. otherwise.

The proposition considers agents with nested partitions, such that agents with finer partitions have a survival index at most as large as those with coarser partitions. In such an economy, an agent survives regardless of the value of his survival index as long as his unawareness is relevant in the limit. So does agent $\tilde{k}-1$ with the coarsest partition, whose unawareness is irrelevant in the limit. Agents with finer partitions whose unawareness is irrelevant in the limit, survive a.s. if their discount factor and beliefs match those of $\tilde{k}-1$. This result is interesting, because it shows that agents with finer partitions can survive even when their survival index is not maximal in the economy. This requires however the presence of agents who are unaware of some states, which in fact lead to different endowment even in the limit. In such a scenario, the partially aware agents cannot consume the entire endowment of the economy: such a consumption stream would not be measurable w.r.t. their awareness partition. Hence, it is the agents with lower survival indexes, but finer partitions who ensure that the markets clear. They consume the 'leftovers' of the partially aware agents and, thus, the fact that unawareness is relevant ensures that they survive a.s..

Moreover, since the comparison of agents' beliefs is restricted to common elements of their partitions, an agent with a finer partition can survive in the presence of agents with coarser partitions and correct beliefs, even if his beliefs about the finer contingencies (disregarded by the others) are wrong. For example, in Example 3, Bob's unawareness is relevant in the limit, whereas Ann's is not. When only Ann and Bob are present in the economy, given equal discount factors, Ann will survive if she and Bob assign equal probabilities to the events $w_1^B = \{s_1; s_3\}$ and $w_2^B = \{s_2; s_4\}$, regardless of whether her estimates about the probabilities of the individual states s_1, s_2, s_3 or s_4 are correct.

Our last two propositions in this section examine an economy, in which the agents' partitions are not necessarily nested. The economy with four agents discussed in Example 3 is an example of such a situation. In this economy, the awareness partitions of Bob and Clara are non-nested / overlapping. We now provide a formal definition of economies with non-nested partitions:

Definition 12 *Agents i and j have non-nested partitions if there are states s, s', s'', $s''' \in S$ such that s, $s' \in w^i$, $s'' \in w^{i\prime}$ and $s''' \in w^{i\prime\prime}$ for some $w^{i\prime} \neq w^{i\prime\prime}$ and $s \in w^j$, $s' \in w^{j\prime}$, s'', $s''' \in w^{j\prime\prime}$ for some $w^j \neq w^{j\prime}$.*

If the states s and s' satisfy this definition, then we will say that i can distinguish between s and s', whereas j cannot.

We will say that agents in the economy have non-nested partitions if, for each agent i, there are states s and $s' \in S$ that i can distinguish, but that no other agent in the economy can distinguish. Formally, there are w^i and $w^{i\prime} \in W^i$, $w^i \neq w^{i\prime}$ such that $s \in w^i$, $s' \in w^{i\prime}$ and for all $j \neq i$, s, $s' \in w^j$ for some $w^j \in W^j$.

Our first result shows that whenever an agent is the only one in the economy capable of distinguishing some relevant contingencies, he survives regardless of his beliefs and discount factor, and regardless of the awareness partitions of the other agents.

Proposition 13 *Consider an economy with coarse contingencies and assume that there exists an agent j and states s and s' such that the partitions of j and any other agent $i \in I \backslash \{j\}$ are non-nested and j can distinguish between s and s', whereas i cannot. Assume that condition (4) holds for s, s' and all $i \in I \backslash \{j\}$. Then agent j survives a.s..*

In particular, in an economy with non-nested partitions as in Definition 12, in which condition (4) holds for any two states s and s' w.r.t. which the partitions of two agents are non-nested, all agents survive a.s..

A special case of Proposition 13 is that in which the finest partition that is coarser than all awareness partitions $\left(\Omega^i\right)_{i \in I}$ is the trivial partition. In this case, no trade occurs in equilibrium and the agents consume their initial endowment streams. The absence of trade does not depend on agents' beliefs or their discount factors. All agents thus survive a.s. regardless of the values of their survival indices.

Our last result introduces a fully aware agent with correct beliefs into the economy from Proposition 13. The presence of such an agent will cause all agents with coarser partitions and incorrect beliefs or lower discount factors to vanish a.s. However, we show that as long as the partially aware agents have non-nested partitions, correct beliefs and discount factors identical to that of the fully aware agent, they survive a.s..

Proposition 14 *Take an economy with coarse contingencies and a set of agents $I' \cup \{j\}$ $(j \notin I')$ Suppose that the condition of non-nested partitions as in Definition 12 is satisfied for the set of agents I and condition (4) holds for any two states s and s' such that at least one agent in I' can distinguish between s and s' and at least one agent cannot. Let j be a fully aware agent. Suppose that all agents have identical discount factors and correct beliefs. Then all agents survive a.s.*

The results of this section have shown that markets do not select specifically for more aware agents. However, when agents differ w.r.t. their beliefs and discount factors, more aware agents have an advantage in that they can survive even when their beliefs are incorrect and their discount factor is smaller than that of less aware agents. This implies that economies with differential awareness can exhibit limited risk sharing, biased state prices and lower saving rates as compared to economies with identical awareness across agents. When the awareness partitions are non-nested, agents with different perceptions of the state space survive. This can reduce the amount of trade in the economy.

The next section discusses two extensions of the model presented above. First, we consider the introduction of financial assets. Next, we analyze the impact of an increase in the awareness of an agent on his survival.

5 Extensions

In this section, we discuss two of the restrictive features of the model presented and analyzed so far. The first concerns the fact that consumption streams are traded directly, rather than through a financial market. The second is related to the fact that our model so far did not allow agents' awareness to increase over time.

5.1 Introducing Assets

In the classic tradition of Arrow and Debreu (1954), the model presented so far has assumed that agents can trade on every possible contingency, provided that their consumption streams remain measurable w.r.t. their awareness partitions. In reality, this type of trade usually occurs via asset markets. While with fully aware agents, one can easily rewrite the model presented above as one with a full set of Arrow securities, this equivalence fails once partial awareness is considered.

A partially aware agent will have a limited understanding of an Arrow security which pays off in a contingency he cannot imagine. It appears natural to assume that a partially aware agent will hold only assets with payoffs measurable w.r.t. his awareness partition. However, as we illustrate below in an example, such a measurability requirement restricts the set of possible trades in an otherwise complete market.

In general, introducing a full set of Arrow securities will fail to 'complete' the market in the sense of allowing all possible trades. However, for some of the cases discussed in Section 4, we will provide a method that allows us to construct an 'effectively complete' set of securities for given awareness partitions of the agents. The initial endowments of the agents can then be translated into initial endowments with such securities so that the resulting equilibrium replicates the equilibrium in the economy, in which consumption streams are traded directly. Hence, the main survival results derived in Section 4 extend to markets that are effectively complete, given the awareness partitions of the agents.

We first use the economy from Example 3 to show how the equilibrium in an economy, in which consumption is traded directly can be replicated by an equilibrium in a market with an appropriately chosen asset structure.

Example 15 *Consider the economy from Example 3 with Ann and Bob. Let $c^A(s)$, $s \in S$ and $c^B(s_1; s_3)$, $c^B(s_2; s_4)$ denote the equilibrium consumption streams in this economy.*

We would now like to introduce assets into the economy such that, by trading in assets, agents can replicate their equilibrium consumption streams. Suppose first that the economy has a full set of Arrow securities. While Ann could use these assets to obtain any consumption stream, Bob cannot conceive of any of the Arrow securities, since their payoff structure violates the measurability requirement and thus, could not trade in them. This, however, would preclude trade between Ann and Bob.

Consider instead, an alternative set of assets, which we can call 'effectively complete'. This set consists of: a security A_{13} that pays 1 in the event $\{s_1; s_3\}$ and nothing otherwise; a security A_{24} that pays 1 in the event $\{s_2; s_4\}$ and nothing otherwise; and the four standard Arrow securities A_1, A_2, A_3 and A_4–

one for each of the states s_1, s_2, s_3 and s_4. From the point of view of both agents Ann and Bob, this is a complete market, in which each of them can achieve any combination of payoffs across the contingencies they perceive. While Ann can trade in all 6 of the available assets, Bob is constrained to trades involving A_{13} and A_{24}.

With this asset structure, we can now reproduce the equilibrium in which consumption is traded directly. To do so, reformulate the initial endowments of the agents in terms of asset holdings: $\bar{a}_1^A = \bar{a}_4^A = 2$, $\bar{a}_2^A = \bar{a}_3^A = 1$, $\bar{a}_{13}^A = \bar{a}_{24}^A = 0$, $\bar{a}_{13}^B = 1$, $\bar{a}_{24}^B = 2$, $\bar{a}_k^B = 0$, $k \in \{1...4\}$, where $\bar{a}_k^i$ denotes the initial holdings of asset A_k by agent $i \in \{A; B\}$. Note that the initial endowments with assets simply replicate the initial endowment of the agents w.r.t. their awareness partitions. The measurability requirement now restricts Bob's portfolio to have $a_\omega^B = 0$ for all $\omega \in \{\{s_1\};\{s_2\};\{s_3\};\{s_4\}\}$. Hence, to reproduce the equilibrium derived above, Ann will have to sell short $c^B(s_1; s_3)$ units of asset A_{13} and $c^B(s_2; s_4)$ units of A_{24}. In fact, Ann can replicate A_{13} by using her initial endowment of A_1 and A_3 and replicate A_{24} by using her initial endowment of A_2 and A_4. The resulting equilibrium portfolios are:

	A_1	A_2	A_3	A_4	A_{13}	A_{24}
Ann	2	1	1	2	$-c^{Bob}(s_1; s_3)$	$-c^{Bob}(s_2; s_4)$
Bob	0	0	0	0	$c^{Bob}(s_1; s_3)$	$c^{Bob}(s_2; s_4)$

These portfolios satisfy the measurability requirement.

Below, we will show how this construction can be extended to any economy with nested partitions.

Definition 16 *For the economy described in Section 2, we will call an asset structure $\mathcal{A}$ effectively complete, if every agent i can achieve all consumption streams measurable w.r.t. his awareness partition by holding a portfolio of assets with payoffs measurable w.r.t. his awareness partition.*

The simplest way to construct an effectively complete asset structure is to introduce what we will call a 'generalized Arrow security' for every element in the partition of each agent $i \in I$. That is, for each $i \in I$, each $t \in \mathbb{N}$, and each $\omega_t^i \in \Omega_t^i$, there should be a security $A_{\omega_t^i}$ paying 1 at time t in the event ω_t^i and 0, otherwise. Let $\rho_A : \Omega \to \prod_{t \in \mathbb{N}} \mathbb{R}_+$ denote the payoff of asset A. For each agent i, let $\bar{a}^i\left(A_{\omega_t^i}\right)$ be this individual's initial endowment with asset $A_{\omega_t^i}$. The payoffs of the securities of which the agent has positive initial endowment are measurable w.r.t. his awareness partition. That is, we can write $\rho_{A_{\omega_t^i}} : \Omega^i \to \prod\limits_{t \in \mathbb{N}} \mathbb{R}_+$.

We will allow agent i to trade only in securities he is aware of, that is, securities such that their payoffs are measurable w.r.t. $\left(\Omega^i; \mathcal{F}^i\right)$. More precisely

Definition 17 *The set of assets of which i is aware, $\mathcal{A}^i$ is defined as:*

$$\mathcal{A}^i = \left\{A \in \mathcal{A} \text{ such that the payoff of } A \text{ is measurable w.r.t. } \left(\Omega^i; \mathcal{F}^i\right)\right\}$$

The method of construction of an effectively complete market described here will, in general, produce a greater number of securities than those necessary to span the market. In particular, this will be true whenever there are some fully aware agents and some partially aware agents. From the perspective of a fully aware agent, all assets other than the usual Arrow securities can be replicated with a portfolio built up from the Arrow securities. The generalized Arrow securities required for partially aware agents to perceive a spanning set are therefore redundant. In general, redundant securities give rise to potential arbitrage opportunities. In equilibrium, however, all such opportunities are eliminated.

We will now define, in more detail, an arbitrage-free equilibrium for an economy with nested partitions:

Definition 18 *An equilibrium with differential unawareness and an effectively complete asset structure $\mathcal{A}$ is a portfolio allocation $\left(a^i\right)_{i\in I}$ and an integrable asset price system $\left(q_A\right)_{A\in\mathcal{A}}$ such that each agent i chooses a portfolio consisting only of assets in $\mathcal{A}^i$ so as to maximize his expected utility such that the budget constraint holds:*

$$a^i = \arg\max \left\{ \begin{array}{c} u_i\left(a^i\left(\sigma_0\right)\right) + \sum_{t=1}^{\infty} \beta_i^t \sum_{\omega_t^i \in \Omega_t^i} \pi^i\left(\omega_t^i\right) u_i\left(\sum_{A\in\mathcal{A}^i} a^i\left(A\right)\rho_A\left(\omega_t^i\right)\right) \\ s.t. \sum_{A\in\mathcal{A}^i} q_A a^i\left(A\right) \le \sum_{A\in\mathcal{A}^i} q_A \bar{a}^i\left(A\right) \\ \sum_{A\in\mathcal{A}^i} a^i\left(A\right)\rho_A\left(\omega_t^i\right) \ge 0 \text{ for all } \omega_t^i \in \Omega^i \end{array} \right\}$$

and asset markets clear:

$$\sum_{i=1}^{n} a^i\left(A\right) = \sum_{i=1}^{n} \bar{a}^i\left(A\right) \quad \forall A \in \mathcal{A}$$

We will now show that, for given initial endowments, we can replicate some of the equilibria (and thus also the associated survival results) obtained in Section 4 by an asset market equilibrium. The key idea is to replicate the initial endowments with a portfolio of assets from an effectively complete asset structure.

Proposition 19 *Consider an economy with a maximally aware agent $i \in I$ whose awareness partition Ω^i is finer than that of any other agent. Let $e^1...e^n$ be the agents' initial endowments and let $c^1...c^n$ be the equilibrium consumption streams in this economy. Then there exists an effectively complete asset market structure and endowments with initial asset holdings $\bar{a}^1...\bar{a}^n$ such that the equilibrium consumption in the resulting economy is also given by $c^1...c^n$.*

Proposition 20 *Consider an economy with two agents i and j with non-nested partitions, Ω^i and Ω^j and initial endowments e^i and e^j. If c^i and c^j are the equilibrium consumption streams in the economy, there exists an effectively complete asset structure and endowments with assets a^i, a^j such that the equilibrium consumption in the resulting economy is also given by c^i and c^j.*

Propositions 19 and 20 show that the main results of our analysis in Section 4 will remain unchanged, as long as markets are effectively complete and agents are restricted to hold securities they are aware of. In particular, Proposition 19 includes the economies considered in Propositions 7 and 14 as a special case, whereas Proposition 20 derives the result for the special case of Proposition 13 when $|I| = 2$. In these cases, the introduction of assets into the economy will leave the equilibrium as well as the survival results unchanged. In particular, if the unawareness of an agent is relevant in the limit, that is, if there is an asset in this economy available in positive supply, with different payoffs in states among which the partially aware agent cannot distinguish even as the time goes to infinity, then in equilibrium, this asset will be allocated to an agent with a finer partition. Such an agent will then survive, regardless of how his beliefs and discount factor compare to those of the partially aware agent.

Note, however, that our result does not encompass the case of $|I| > 2$ of Proposition 13. In particular, the example on p. 7 of Quiggin and Siddiqi (2015) shows that with 3 agents with non-nested awareness partitions, an economy with an effectively complete asset structure might exhibit arbitrage opportunities. In fact, the requirement that agents only hold assets with payoffs measurable relative to their awareness structure is much more restrictive than measurability of equilibrium consumption streams. The former imposes restrictions on the specific trades, as well as on the final allocation. When there is no maximally aware trader in the economy (as e.g., in the case of nested partitions), who can insure that all trades are measurable w.r.t. the individual partitions, the asset market equilibrium might differ from the equilibrium with consumption, or might even fail to exist.

5.2 Increasing Awareness

In our analysis so far, we have assumed that trade occurs only once, in period 0 and agents cannot retrade the resulting equilibrium allocation, even if their level of awareness increases with time. In our context, an increase in the level of awareness can be interpreted as a better understanding of the underlying uncertainty. That is, in becoming more aware, a agent learns to distinguish between contingencies that he considered *a priori* identical. In this section, we will allow the level of awareness of a agent to increase with time and will discuss the implications for survival.

For simplicity, we will consider a one-time change in the level of awareness. Formally, assume that at node $\sigma^*_{t^*}$, agent i becomes aware of a (weakly) finer partition of his state space, call this partition, Ω^{*i}. Let W^{*i} be the partition over states corresponding to the increased awareness Ω^{*i} of agent i with a representative element w^{*i}.

Let $\Omega_{\sigma^*_{t^*}}$ denote the set of all (infinite) paths σ^* with initial node $\sigma^*_{t^*}$, and $\Omega^{*i}_{\sigma^*_{t^*}}$ denote the set of all (infinite) paths ω^{*i} with initial node $\omega^{i*}_{t^*}$ such that $\sigma^*_{t^*} \in \omega^{i*}_{t^*}$. Intuitively, $\Omega^{*i}_{\sigma^*_{t^*}}$ is the set of paths i considers possible given his increased level of awareness and given that the economy is in node $\sigma^*_{t^*}$.

Becoming more aware will require the agent to assign probabilities to the finer contingencies. Let i's probability distribution on $\left(\Omega^{*i};\mathcal{F}^{*i}\right)$ be denoted by π^{*i}. We will require that the revised beliefs are consistent with i's initial beliefs in the sense that:

$$\pi^{i}\left(\omega_{t}^{i}\right)=\sum_{\left\{\omega_{t}^{*i}\in\Omega^{*i}\mid\omega_{t}^{*i}\in\omega_{t}^{i}\right\}}\pi^{*i}\left(\omega_{t}^{*i}\right).$$

In particular, the revised beliefs will be consistent while satisfying the i.i.d. property if

$$\begin{aligned}\pi^{*i}\left(w^{*i}\mid\omega_{t}^{*i}\right) &= \pi^{*i}\left(w^{*i}\right)\text{ and} \\ \pi^{i}\left(w^{i}\right) &= \sum_{\left\{w^{*i}\in W^{*i}\mid w^{*i}\in w^{i}\right\}}\pi^{*i}\left(w^{*i}\right)\end{aligned}\tag{5}$$

for all ω_{t}^{*i}. Note, however that the beliefs on the finer partition π^{*i} are not uniquely determined by the initial beliefs.

At $\sigma_{t^*}^{*}$, agents might want to reoptimize taking into account this finer partition. To compute the new equilibrium at $\sigma_{t^*}^{*}$, we will take the initial equilibrium allocation to be the agents' initial endowment and will compute the new allocation and the new equilibrium prices for the economy that starts at $\sigma_{t^*}^{*}$, taking into account that the consumption of agent i now has to be measurable w.r.t. the finer partition Ω^{*i}. To make sure that this reoptimization is meaningful, we will make the following assumption:

Assumption 4 Either all agents in the economy are aware of $\sigma_{t^*}^{*}$, or the agents' partitions are nested, at least one of the agents in the economy is aware of $\sigma_{t^*}^{*}$ and, for each $k\in I$ who is unaware of $\sigma_{t^*}^{*}$ and every $\sigma_t\in\omega_t^{*k}\backslash\Omega_{\sigma_{t^*}^{*}}$, there is a $\sigma_t'^{*}\in\omega_t^{*k}\cap\Omega_{\sigma_{t^*}^{*}}$ such that $e\left(\sigma_t'^{*}\right)\leq e\left(\sigma_t\right)$.

Consider, for example, the economy from Example 3 consisting of *Ann* and *Bob* and assume that endowments are i.i.d. over time. While there is no node besides σ_0 of which both agents are aware, at every node σ_t, *Ann* is aware of the node. Furthermore, since the economy is i.i.d., we know that if $\sigma_t\in\omega_t^{*B}\backslash\Omega_{\sigma_{t^*}^{*}}$ and $\sigma_t^{*\prime}\in\omega_t^{*B}\cap\Omega_{\sigma_{t^*}^{*}}$, then $s_t\in\{s_1;s_3\}$ iff $s_t^{*'}\in\{s_1;s_3\}$ (and $s_t\in\{s_2;s_4\}$ iff $s_t'^{*}\in\{s_2;s_4\}$). In the former case, $e\left(\sigma_t\right)\in\{2;3\}$ and choosing $\sigma_t^{*\prime}=\left(\sigma_{t-1}^{*\prime};s_3\right)$ implies $e\left(\sigma_t\right)\geq\min\{2;3\}=e\left(\sigma_t'^{*}\right)$. In the latter, $e\left(\sigma_t\right)\in\{3;4\}$ and choosing $\sigma_t^{*\prime}=\left(\sigma_{t-1}^{*\prime};s_2\right)$ implies $e\left(\sigma_t\right)\geq\min\{4;3\}=e\left(\sigma_t'^{*}\right)$. Hence, in this economy every node satisfies the condition imposed on $\sigma_{t^*}^{*}$.

Remark 21 *Consider an economy with i.i.d. endowments, that is* $e\left(\sigma_t;s\right)=e\left(\sigma_{t'}';s\right)$ *for all* σ_t, $\sigma_{t'}'\in\Omega$. *It will satisfy Assumption 4 for every* $\sigma_{t^*}^{*}$.

Definition 22 *Let* $\left(c^i\right)_{i\in I}$ *be an equilibrium allocation of the economy with coarse contingencies. An equilibrium with increased awareness* $\left(\Omega^{*i}\right)_{i\in I}$ *at* $\sigma_{t^*}^{*}$ *is an integrable price system* $\left(p^*\left(\sigma_t\right)\right)_{\sigma_t\in\Omega^*}$ *and consumption streams* $\left(c^{*i};\left(c^j\right)_{j\neq i}\right)$

defined on Ω such that (i) all consumers i are maximizing their expected utility subject to their budget constraint and awareness partition and (ii) markets clear:

$$c^{*i} = \arg\max_{c^{*i}} \left\{ \begin{array}{c} u_i \left(c^i \left(\sigma^*_{t^*} \right) \right) + \sum_{t=t^*}^{\infty} \beta_i^t \sum_{\omega_t^{*i} \in \Omega^{*i}_{\sigma^*_{t^*}}} \pi^{*i} \left(\omega_t^{*i} \right) u_i \left(c^{*i} \left(\omega_t^{*i} \right) \right) \\ \text{s.t. } \sum_{t \geq t^*} \sum_{\omega_t^{*i} \in \Omega^{*i}_{\sigma^*_{t^*}}} \sum_{\sigma^*_t \in \omega_t^{*i}} p^* \left(\sigma_t \right) c^{*i} \left(\omega_t^{*i} \right) \\ \leq \sum_{t \geq t^*} \sum_{\omega_t^{i} \in \Omega^{i}_{\sigma^*_{t^*}}} \sum_{\sigma_t \in \omega_t^{i}} p^* \left(\sigma_t \right) c^{i} \left(\omega_t^{i} \right) \end{array} \right\}$$

$$\sum_{i=1}^{I} c^{*i} \left(\sigma_t \right) = \sum_{i=1}^{I} c^{i} \left(\sigma_t \right) \quad \forall \sigma_t \in \Omega^*_{\sigma^*_{t^*}}.$$

Proposition 23 *Under Assumptions 1–4, an equilibrium with increased awareness exists. Furthermore, in such an equilibrium $p\left(\sigma_t\right) = 0$ for all $\sigma_t \notin \Omega_{\sigma^*_{t^*}}$. Hence, the equilibrium consumption can be characterized by the f.o.c.:*

$$\frac{u_i' \left(c^i \left(\omega_t^{*i} \right) \right)}{\beta_i \pi^i \left(\omega_{t+1}^{*i} \mid \omega_t^{*i} \right) u_i' \left(c^i \left(\omega_{t+1}^{*i} \right) \right)} = \frac{p^* \left(\omega_t^{*i} \right)}{p^* \left(\omega_{t+1}^{*i} \right)} =: \frac{\sum_{\sigma^*_t \in \omega_t^{*i} \cap \Omega_{\sigma^*_{t^*}}} p^* \left(\sigma^*_t \right)}{\sum_{\sigma^*_{t+1} \in \omega_{t+1}^{*i} \cap \Omega_{\sigma^*_{t^*}}} p^* \left(\sigma^*_{t+1} \right)}.$$

We can now discuss the survival of an agent whose awareness has increased. As we know from Section 4, given identical discount factors, the revised beliefs π^{*i} will play a crucial role for i's survival. The next result follows directly from Proposition 11:

Corollary 24 *Suppose that the economy satisfies Assumptions 1–3. Consider a population of agents with nested partitions Ω^1 strictly finer than Ω^2... strictly finer than Ω^n, such that agent 2's unawareness is relevant in the limit. Let $\sigma^*_{t^*} \in \Omega$ satisfy Assumption 4. Suppose that at $\sigma^*_{t^*}$ agent $i > 1$'s awareness increases to Ω^{*i}, such that the new set of partitions satisfies the nested property. If all agents have correct beliefs and identical discount factors, then*

1. *if i's revised probability distribution π^{*i} satisfies (5) and is correct, then i survives a.s.;*
2. *if i's revised probability distribution π^{*i} satisfies (5) and is incorrect, then i vanishes a.s..*

To understand the result, note that three cases are possible. First, i's new level of awareness can be the finest in the population. However, since the unawareness of agent 1 is not relevant in the economy, agent i can survive iff he adopts correct beliefs on the finer partition. Second, i's new awareness partition could coincide with Ω^1 . In this case, since agent 1 has correct beliefs, i can survive iff he also adopts correct beliefs. Finally, if Ω^{*i} is coarser than Ω^1, then we can use Proposition 9 to conclude that i will disappear relative to agent 1 unless i adopts correct beliefs.

We thus conclude that an agent who has become more aware will only survive if he simultaneously adopts correct beliefs on the finer partition. However, we will now show that becoming more aware makes it "harder" to form correct beliefs, in a sense we will make precise. Recall that for a partially aware agent i with a state partition W^i to have correct beliefs, it is necessary and sufficient that $\pi^i\left(w^i\right) = \sum_{s \in w^i} \pi(s)$ for all $w^i \in W^i$. In particular, a fully unaware agent, for whom $W^i = \{\{s \in S\}\}$ trivially has correct beliefs and survives.

Denoting by $\Delta^{|S|-1}$ the simplex of all probability distributions over the states in S, let

$$\mathcal{P}\left(W^i\right) = \left\{\tilde{\pi} \in \Delta^{|S|-1} \mid \tilde{\pi}\left(w^i\right) = \sum_{s \in w^i} \pi(s) \text{ for all } w^i \in W^i\right\}$$

be the set of all probability distributions over S the restriction of which to W^i implies that an agent with this partition has correct beliefs. Note that $\left(\pi(s)_{s \in S}\right) \in P\left(W^i\right)$ always holds, and whenever $W^i \neq \left\{\{s\}_{s \in S}\right\}$, $P\left(W^i\right)$ is not a singleton. Furthermore, we have:

Lemma 25 *For any W^{*i} strictly finer than W^i,*

$$\mathcal{P}\left(W^{*i}\right) \subset \mathcal{P}\left(W^i\right)$$

and $P\left(W^{*i}\right)$ has a Lebesgue measure 0 relative to $P\left(W^i\right)$.

It is in that sense that for an agent whose awareness increases adopting correct beliefs on the new partition might not be straightforward: unless the agent has a good understanding of the underlying uncertainty, guessing the correct distribution within the set $P\left(W^i\right)$ is a 0-probability event. Hence, even though an increase in agent's awareness increases his welfare as long as his beliefs remain correct (see Proposition 8), the requirement that the revised beliefs be correct is hardly innocuous.

We next consider an agent who adopts a prior over a set of possible distributions and tries to use Bayesian updating to learn the correct one. In this case, the agent's beliefs will no longer satisfy the i.i.d. property. We can use the result of Blume and Easley (2006) to show that in general, an increase in awareness combined with Bayesian learning of the probabilities will not allow the agent to survive.

Suppose that upon becoming aware of Ω^{*i}, agent i adopts a set of "models" Θ^{*i}, that is, a set of i.i.d. processes on Ω^{*i} containing the true process. Each element of Θ^{*i} is thus a probability distribution $\left(\theta\left(w^{*i}\right)\right)_{w^{*i} \in W^{*i}}$ and there is a true model, $\theta^{true} \in \Theta^{*i}$ such that $\theta^{true}\left(w^{*i}\right) =: \pi\left(w^{*i}\right)$ for all $w^{*i} \in W^{*i}$. We will assume that the set of models Θ^{*i} is open and bounded and thus, satisfies Assumption 5 of Blume and Easley (2006). We then obtain the following Corollary to Blume and Easley (2006), Theorem 5:

Corollary 26 *Suppose that the economy satisfies Assumptions 1–3. Consider a population of agents with nested partitions Ω^1 strictly finer than Ω^2... strictly* *finer than Ω^n. Let $\sigma^*_{t^*} \in \Omega$ satisfy Assumption 4. Suppose that at $\sigma^*_{t^*}$ agent $i < 1$'s awareness increases to $\Omega^{*i} = \Omega^{i-1}$. If all agents have correct beliefs and identical discount factors and if i adopts a prior on the set of models Θ^{*i} which is absolutely continuous w.r.t. the Lebesgue measure and updates it according to Bayesian rule, i vanishes a.s. relative to $i-1$.*

We thus obtain a 'paradox of ignorance', similar to that attributed to Socrates.[7] Even though an increase in awareness leads to an increase in the agent's welfare, from the point of view of survival, having partial knowledge about the world might be preferable to being a Bayesian who is assigning probabilities in an ill-informed way. More generally, we have shown that while markets with more aware agents provide more opportunities for risk-sharing, they also pose greater risk for the survival of traders who might misjudge probabilities.

6 Concluding Comments

The standard model of financial markets is one in which all agents are rational and this fact is common knowledge. Assuming common priors, no agents can consciously disagree (Aumann 1976). It is natural, therefore to consider whether the assumption of common priors can be modified. The analysis of Blume and Easley shows that, if all agents are rational, in the sense that they are aware of all possible contingencies, then differences in prior beliefs are, ultimately, irrelevant since only agents with correct beliefs will survive.

We have shown, by contrast, that the presence of differential awareness allows for the survival of agents with both differing beliefs and differing awareness. On the one hand, more aware agents may survive, even when their beliefs are less accurate than those of others. Conversely, less aware agents (those with a coarser partition of the state space) will survive if their beliefs regarding the coarser state space that they perceive are accurate. Moreover, the cognitive and information requirements to form accurate beliefs about a coarse partition of the state space are less demanding than the requirements for accurate probabilities regarding the full set of economically relevant states. In particular, agents with non-stochastic endowments and minimal awareness who invest only in bonds, will survive a.s., though they will forgo consumption opportunities available from insuring others.

This analysis leaves open the question of how differences in prior beliefs should be modelled. One possibility is that agents display reduction unawareness in the sense of Grant and Quiggin (2014) in the sense that they fail to consider some possible contingencies. Differential awareness in the sense of reduction will, in general, give rise to different prior beliefs regarding the reduced state space under consideration. These issues are addressed in a companion paper (Guerdjikova and Quiggin 2016).

[7] On being told that an oracle had named him the wisest of the Greeks, Socrates is said to have replied that his wisdom consisted of knowing that he knew nothing.

7 Appendix

Proof of Proposition 2:

An equilibrium of the economy exists under the following conditions (Bewley 1972):

1. the consumption sets are convex, Mackey-closed and contained in the set of essentially bounded measurable functions;

2. the preferences of the agents are complete and transitive;

3. the better sets are convex and Mackey-closed;

4. the worse sets are closed in the norm topology;

5. there exists a set of paths with strictly positive measure such that the preferences of all agents satisfy strict monotonicity on this set, that is adding a constant to the payoff in each state and each period makes the agent strictly better off; and

6. for all agents, the initial endowments are in the interior of the consumptions sets.

We can assume that the consumption set of a agent $i \in \{1 \dots I\}$ is given by the sets of all essentially bounded measurable functions on Ω^i and, hence, satisfies condition 1. We can then define the function $V_0^i\left(c^i\right)$ on the set of all essentially bounded measurable functions on Ω, while the measurability restriction w.r.t. Ω^i is imposed by the consumption set of i. Condition 2 is then trivially satisfied. The convexity requirement in condition 3 follows from the concavity of the utility function u^i. Further, V_0^i is uniformly continuous and, hence, continuous w.r.t. the Mackey topology. This means that both the better and the worse sets are closed w.r.t. the Mackey topology, and, hence, also in the norm topology. The second requirement in conditions 3 and 4 are therefore satisfied.

For condition 5, and an agent i, take the set of paths to be Ω. Note that V_0^i is monotonic. Take any consumption stream c. Adding a positive amount to c strictly improves the act. Hence, the preferences of all agents are strictly monotonic on Ω.

Finally, Assumption 2 ensures that the endowment stream of each agent is uniformly bounded away from 0 and from infinity, and is, therefore, in the interior of this agent's consumption set, thus implying condition 6 We conclude that an equilibrium of the economy exists.

Note that the measurability condition on i's consumption ensures that

$$u_i'\left(c^i\left(\sigma_t\right)\right) = u_i'\left(c^i\left(\sigma_t'\right)\right)$$

for all σ_t, $\sigma_t' \in \Omega_t^i$. If $p(\cdot)$ is an equilibrium price system, then condition (2)

$$\frac{u_i'\left(c^i\left(\omega_t^i\right)\right)}{\beta_i \pi^i\left(\omega_{t+1}^i \mid \omega_t^i\right) u_i'\left(c^i\left(\omega_{t+1}^i\right)\right)} = \frac{p\left(\omega_t^i\right)}{p\left(\omega_{t+1}^i\right)} = \frac{\sum_{\sigma_t \in \omega_t^i} p\left(\sigma_t\right)}{\sum_{\sigma_{t+1} \in \omega_{t+1}^i} p\left(\sigma_{t+1}\right)}$$

is the first-order condition of agent i's maximization problem at state σ_t. Hence, it will be satisfied in any equilibrium in which agent i chooses an interior allocation on all finite paths with positive probabilities. We now show that Assumptions 1–3 imply that the optimal consumption streams of all agents will be strictly positive on all finite paths which have positive probability. To show this, we demonstrate that the marginal rate of substitution between consumption at σ_0 and ω_t will always be strictly positive and finite, provided that the true probability of ω_t is positive.

Since the initial endowment is uniformly bounded above, so are all the consumption streams in equilibrium. Hence, by Assumption 1, u_i' is always strictly positive. Furthermore, setting $c(\sigma_0) = 0$ is not optimal, since, by Assumption 2, endowment is uniformly bounded away from 0 and by Assumption 1, $u'(0) = \infty$. Take an arbitrary ω_t^i such that $\pi\left(\omega_t^i\right) > 0$, and hence, by Assumption 3, $\pi^i\left(\omega_t^i\right) > 0$. If $c(\omega_t) = 0$, and if $p(\sigma_0)$, $p(\omega_t) > 0$, an iteration on (2) gives

$$MRS^i\left(c^i(\sigma_0); c^i(\omega_t)\right) = \frac{u_i'\left(c^i(\sigma_0)\right)}{\beta_i^t \pi^i\left(\omega_t^i\right) u_i'\left(c^i\left(\omega_t^i\right)\right)} = 0 < \frac{p(\sigma_0)}{p(\omega_t)},$$

which cannot hold in the optimum. Hence, $c^i(\omega_t) = 0$ can only obtain if $\pi^i(\omega_t) = 0$, or, by Assumption 2, if $\pi(\omega_t) = 0$. We thus obtain that i will have strictly positive consumption on all finite paths which have positive probability w.r.t. the truth. This, in turn implies that the first order condition will hold on all such paths.

Derivations for Example 3:

Claim 1: Neither of the two agents is insured against idiosyncratic risk in equilibrium.

Proof of Claim 1:

Let $u_A(\cdot)$ and $u_B(\cdot)$ be A's and B's concave von Neumann–Morgenstern utility. Standard expected utility maximization then gives the f.o.c's for Ann:

$$\frac{u_A'\left(c^A(s)\right)\pi(s)}{u_A'\left(c^A(s')\right)\pi(s')} = \frac{p_s}{p_{s'}} \text{ for } s\,,\, s' \in \{1\ldots 4\}$$

and for Bob (since $c^B(s_1) = c^B(s_3)$, $c^B(s_2) = c^B(s_4)$)

$$\frac{u_B'\left(c^B(s_1)\right)(\pi(s_1) + \pi(s_3))}{u_B'\left(c^B(s_2)\right)(\pi(s_2) + \pi(s_4))} = \frac{p_1 + p_3}{p_2 + p_4}.$$

Combining these, we obtain:

$$\begin{aligned}
\frac{u_B'\left(c^B(s_1)\right)(\pi(s_1) + \pi(s_3))}{u_B'\left(c^B(s_2)\right)(\pi(s_2) + \pi(s_4))} &= \frac{p_1}{p_2}\frac{1 + \frac{p_3}{p_1}}{1 + \frac{p_4}{p_2}} = \frac{u_A'\left(c^A(s_1)\right)\pi(s_1)}{u_A'\left(c^A(s_2)\right)\pi(s_2)} \frac{1 + \frac{u_A'\left(c^A(s_3)\right)\pi(s_3)}{u_A'\left(c^A(s_1)\right)\pi(s_1)}}{1 + \frac{u_A'\left(c^A(s_4)\right)\pi(s_4)}{u_A'\left(c^A(s_2)\right)\pi(s_2)}} \\
\frac{u_B'\left(c^B(s_1)\right)(\pi(s_1) + \pi(s_3))}{u_B'\left(c^B(s_2)\right)(\pi(s_2) + \pi(s_4))} &= \frac{u_A'\left(c^A(s_1)\right)\pi(s_1) + u_A'\left(c^A(s_3)\right)\pi(s_3)}{u_A'\left(c^A(s_2)\right)\pi(s_2) + u_A'\left(c^A(s_4)\right)\pi(s_4)}.
\end{aligned}$$

Indeed, in a manner of contradiction, assume that $c^B(s_1) = c^B(s_2)$ and note that this implies:

$$\begin{aligned}
\frac{u_B'\left(c^B(s_1)\right)(\pi(s_1)+\pi(s_3))}{u_B'\left(c^B(s_2)\right)(\pi(s_2)+\pi(s_4))} &= \frac{(\pi(s_1)+\pi(s_3))}{(\pi(s_2)+\pi(s_4))} = \frac{u_A'\left(c^A(s_1)\right)\pi(s_1)+u_A'\left(c^A(s_3)\right)\pi(s_3)}{u_A'\left(c^A(s_2)\right)\pi(s_2)+u_A'\left(c^A(s_4)\right)\pi(s_4)} \\
&= \frac{u_A'\left(3-c^B(s_1)\right)\pi(s_1)+u_A'\left(2-c^B(s_1)\right)\pi(s_3)}{u_A'\left(3-c^B(s_1)\right)\pi(s_2)+u_A'\left(4-c^B(s_1)\right)\pi(s_4)} \\
&> 1\ ,
\end{aligned}$$

since

$$\frac{u_A'\left(3-c^B(s_1)\right)\pi(s_1)+u_A'\left(2-c^B(s_1)\right)\pi(s_3)}{u_A'\left(3-c^B(s_1)\right)\pi(s_2)+u_A'\left(4-c^B(s_1)\right)\pi(s_4)} > \frac{(\pi(s_1)+\pi(s_3))}{(\pi(s_2)+\pi(s_4))}$$

is equivalent to:

$$\begin{aligned}
& u_A'\left(3-c^B(s_1)\right)\pi(s_1)(\pi(s_2)+\pi(s_4))+u_A'\left(2-c^B(s_1)\right)\pi(s_3)(\pi(s_2)+\pi(s_4)) \\
> \; & u_A'\left(3-c^B(s_1)\right)\pi(s_2)(\pi(s_1)+\pi(s_3))+u_A'\left(4-c^B(s_1)\right)\pi(s_4)(\pi(s_1)+\pi(s_3))
\end{aligned}$$

$$\begin{aligned}
&\left[u_A'\left(3-c^B(s_1)\right)-u_A'\left(4-c^B(s_1)\right)\right]\pi(s_1)\pi(s_4)+\left[u_A'\left(2-c^B(s_1)\right)-u_A'\left(4-c^B(s_1)\right)\right]\pi(s_3)\pi(s_4) \\
&\quad +\left[u_A'\left(2-c^B(s_1)\right)-u_A'\left(3-c^B(s_1)\right)\right]\pi(s_2)\pi(s_3) > 0
\end{aligned}$$

which is always satisfied, since u_A' is a decreasing function. We thus obtain a contradiction to the assumption that B is fully insured against idiosyncratic risk in equilibrium.

Claim 2: If

$$\pi(s_1)\pi(s_2)-\pi(s_3)\pi(s_4) \le 0 \tag{6}$$

A's expected consumption is higher than her expected initial endowment.

Proof of Claim 2:

From the fact that B's utility function is concave and thus, B partially insures against risk, it follows that the equilibrium consumption of A satisfies: $c^A(s_1) < 2$, $c^A(s_2) > 1$, $c^A(s_3) < 1$, $c^A(s_4) > 2$ with

$$\begin{aligned}
c^A(s_4) &= 4-c^B(s_2) > c^A(s_1) = 3-c^B(s_1) > \\
&> c^A(s_2) = 3-c^B(s_2) > c^A(s_3) = 2-c^B(s_1)
\end{aligned}$$

From A's f.o.c. we then conclude that the equilibrium prices satisfy:

$$\frac{p_4^*}{\pi(s_4)} < \frac{p_1^*}{\pi(s_1)} < \frac{p_2^*}{\pi(s_2)} < \frac{p_3^*}{\pi(s_3)}\ .$$

Suppose to the contrary of Claim 2 that

$$E_\pi\left[c^A(s)\right] < E_\pi\left(e^A(s)\right) = 2\pi(s_1)+\pi(s_2)+\pi(s_3)+2\pi(s_4)\ ,$$

and, hence,

$$\begin{aligned}&\left[3-c^{B}\left(s_{1}\right)\right] \pi\left(s_{1}\right)+\left[2-c^{B}\left(s_{1}\right)\right] \pi\left(s_{3}\right)+\left[3-c^{B}\left(s_{2}\right)\right] \pi\left(s_{2}\right) \\ &+\left[4-c^{B}\left(s_{2}\right)\right] \pi\left(s_{4}\right) \leq 2 \pi\left(s_{1}\right)+\pi\left(s_{2}\right)+\pi\left(s_{3}\right)+2 \pi\left(s_{4}\right),\end{aligned}$$

or

$$c^{B}\left(s_{2}\right) \geq 2+\frac{\pi\left(s_{1}\right)+\pi\left(s_{3}\right)}{\pi\left(s_{2}\right)+\pi\left(s_{4}\right)}\left[1-c^{B}\left(s_{1}\right)\right]$$

It follows that:

$$\begin{aligned}&E_{\pi} u_{A}\left(c^{A}\right)=\pi\left(s_{1}\right) u_{A}\left(3-c^{B}\left(s_{1}\right)\right)+\pi\left(s_{3}\right) u_{A}\left(2-c^{B}\left(s_{1}\right)\right)+\pi\left(s_{2}\right) u_{A}\left(3-c^{B}\left(s_{2}\right)\right) \\ &+\pi\left(s_{4}\right) u_{A}\left(4-c^{B}\left(s_{2}\right)\right) \leq \pi\left(s_{1}\right) u_{A}\left(2+\left(1-c^{B}\left(s_{1}\right)\right)\right)+\pi\left(s_{3}\right) u_{A}\left(1+\left(1-c^{B}\left(s_{1}\right)\right)\right) \\ &+\pi\left(s_{2}\right) u_{A}\left(1-\frac{\pi\left(s_{1}\right)+\pi\left(s_{3}\right)}{\pi\left(s_{2}\right)+\pi\left(s_{4}\right)}\left(1-c^{B}\left(s_{1}\right)\right)\right)+\pi\left(s_{4}\right) u_{A}\left(2-\frac{\pi\left(s_{1}\right)+\pi\left(s_{3}\right)}{\pi\left(s_{2}\right)+\pi\left(s_{4}\right)}\left(1-c^{B}\left(s_{1}\right)\right)\right)\end{aligned}$$

$$\begin{aligned}< \quad & \left(\pi\left(s_{1}\right)+\pi\left(s_{4}\right)\right) u_{A}\left(2+\left(1-c^{B}\left(s_{1}\right)\right)\left(\frac{\pi\left(s_{1}\right) \pi\left(s_{2}\right)-\pi\left(s_{3}\right) \pi\left(s_{4}\right)}{\left(\pi\left(s_{2}\right)+\pi\left(s_{4}\right)\right)\left(\pi\left(s_{1}\right)+\pi\left(s_{4}\right)\right)}\right)\right) \\ & +\left(\pi\left(s_{2}\right)+\pi\left(s_{3}\right)\right) u_{A}\left(1+\left(\frac{\pi\left(s_{3}\right) \pi\left(s_{4}\right)-\pi\left(s_{1}\right) \pi\left(s_{2}\right)}{\left(\pi\left(s_{2}\right)+\pi\left(s_{4}\right)\right)\left(\pi\left(s_{2}\right)+\pi\left(s_{3}\right)\right)}\right)\left(1-c^{B}\left(s_{1}\right)\right)\right)\end{aligned}$$

Since $c^{B}\left(s_{1}\right)>1$, for

$$\pi\left(s_{1}\right) \pi\left(s_{2}\right)-\pi\left(s_{3}\right) \pi\left(s_{4}\right) \leq 0,$$

this is a mean-preserving spread of the initial endowment and we have

$$E_{\pi} u_{A}\left(c^{A}\right)<E_{\pi} u_{A}\left(e^{A}\right),$$

in contradiction to utility maximization.

Claim 3: It is impossible to ensure all agents in the economy against idiosyncratic risk.

Proof of Claim 3:

Indeed, suppose to the contrary that $c^{i}\left(s_{1}\right)=c^{i}\left(s_{2}\right)$ for all agents $i \in\{A ; B ; C ; D\}$. For A to be fully insured across s_{1} and s_{2}, we need:

$$\frac{u_{A}^{\prime}\left(c^{A}\left(s_{1}\right)\right)}{u_{A}^{\prime}\left(c^{A}\left(s_{2}\right)\right)}=1=\frac{p_{1}}{p_{2}}.$$

Furthermore, since C and D are fully insured across s_{1} and s_{2}, the measurability requirement on their consumption implies that they are fully insured across all states:

$$c^{i}\left(s_{1}\right)=c^{i}\left(s_{2}\right)=c^{i}\left(s_{3}\right)=c^{i}\left(s_{4}\right), i \in\{C ; B\},$$

and hence,

$$\frac{u_{B}^{\prime}\left(c^{B}\left(s_{3}\right)\right)}{u_{B}^{\prime}\left(c^{B}\left(s_{4}\right)\right)}=\frac{p_{1}+p_{3}}{p_{2}+p_{4}},$$

or $p_3 = p_4$. But this would imply that

$$\frac{u_i'\left(c^i\left(s_3\right)\right)}{u_i'\left(c^i\left(s_4\right)\right)} = \frac{p_3}{p_4} = 1 ,$$

or that both A and D have to be fully insured across states s_3 and s_4, which is impossible.

Claim 4: In general, A and D will not be fully insured against idiosyncratic risk in equilibrium.

Proof of Claim 4:

To give an example, suppose that $u_i(c) = \ln c$ for $i \in \{A; B; D\}$, whereas $u_C(c) = c^{\frac{1}{2}}$. Suppose that A and D were fully insured against idiosyncratic risk, then by the same argument as above $p_1 = p_2 =: p$. Since B and C are risk-averse, they will try to smooth consumption across the states they perceive. Hence, in equilibrium, their consumption will satisfy:

$$\begin{aligned} 2 &> c^B\left(s_2\right) = c^B\left(s_4\right) > c^B\left(s_1\right) = c^B\left(s_3\right) > 1 \\ 2 &> c^C\left(s_1\right) = c^C\left(s_4\right) > c^C\left(s_2\right) = c^C\left(s_3\right) > 1 \end{aligned}$$

But then,

$$c^A\left(s_1\right) = \frac{6 - c^B\left(s_1\right) - c^C\left(s_1\right) - c^D\left(s_1\right)}{2} = \frac{6 - c^B\left(s_2\right) - c^C\left(s_2\right) - c^D\left(s_1\right)}{2} = c^A\left(s_2\right)$$

and we obtain

$$c^B\left(s_1\right) + c^C\left(s_1\right) = c^B\left(s_2\right) + c^C\left(s_2\right) . \tag{7}$$

The demand functions of B and C satisfy:

$$\begin{aligned} c^B\left(s_1\right) &= c^B\left(s_3\right) = \frac{3p + p_3 + 2p_4}{2\left(p_3 + p\right)} \\ c^B\left(s_2\right) &= c^B\left(s_4\right) = \frac{3p + p_3 + 2p_4}{2\left(p_4 + p\right)} \end{aligned}$$

$$\begin{aligned} c^C\left(s_2\right) = c^C\left(s_3\right) &= \frac{3p + p_3 + 2p_4}{2p + p_3 + p_4} \frac{\left(p + p_4\right)}{\left(p + p_3\right)} \\ c^C\left(s_1\right) = c^C\left(s_4\right) &= \frac{3p + p_3 + 2p_4}{2p + p_3 + p_4} \frac{\left(p + p_3\right)}{\left(p + p_4\right)} \end{aligned}$$

and substituting into (7), we obtain:

$$\frac{3p + p_3 + 2p_4}{2\left(p_3 + p\right)} + \frac{3p + p_3 + 2p_4}{2p + p_3 + p_4} \frac{\left(p + p_3\right)}{\left(p + p_4\right)} = \frac{3p + p_3 + 2p_4}{2p + p_3 + p_4} \frac{\left(p + p_4\right)}{\left(p + p_3\right)} + \frac{3p + p_3 + 2p_4}{2\left(p_4 + p\right)}$$

W.l.o.g., we can normalize $3p + p_3 + 2p_4 = 1$ and simplify to:

$$2p\left(p_3 - p_4\right) = \left(p_4 - p_3\right)\left(p_4 + p_3\right) .$$

Since the only solution of this equation is $p_3 = p_4$, it follows that

$$\begin{aligned} c^A(s_3) &= c^A(s^4) \\ c^D(s_3) &= c^D(s^4) \\ c^B(s_3) &= c^B(s_4) \\ c^C(s_3) &= c^C(s_4), \end{aligned}$$

in contradiction to the existence of aggregate risk.

To simplify the proofs of the following results, we state and prove the following:

Lemma 27 *Consider two agents i and j such that j is fully aware and i is partially aware. In equilibrium, for any path ω^i,*

$$\begin{aligned} &\lim_{T\to\infty} \frac{1}{T+1} \ln \frac{u_i'\left(c^i\left(\omega^i_{T+1}\right)\right)}{\left(\sum_{\tilde{\sigma}_{T+1}\in\omega^i_{T+1}} u_j'\left(c^j\left(\tilde{\sigma}_{T+1}\right)\right)\pi^j\left(\tilde{\sigma}_{T+1}\mid\omega^i_{T+1}\right)\right)} \\ = &\lim_{T\to\infty} \ln\frac{\beta_j}{\beta_i} + \left(\sum_{w^i}\pi\left(w^i\right)\ln\frac{\pi\left(w^i\right)}{\pi^i\left(w^i\right)} - \sum_{w^i}\pi\left(w^i\right)\ln\frac{\pi\left(w^i\right)}{\pi^j\left(w^i\right)}\right). \end{aligned}$$

<u>Proof of Lemma 27:</u>

We will use the analogue of the Blume and Easley (2006) decomposition. Applying condition (2) to two agents, i, who is partially aware and j, who is fully aware gives:

$$\frac{u_i'\left(c^i\left(\sigma_0\right)\right)}{\beta_i u_i'\left(c^i\left(\omega^i_{T+1}\right)\right)\pi^i\left(\omega^i_{T+1}\right)} = \frac{u_j'\left(c^j\left(\sigma_0\right)\right)}{\beta_j\sum_{\tilde{\sigma}_{T+1}\in\omega^i_{T+1}} u_j'\left(c^j\left(\tilde{\sigma}_{T+1}\right)\right)\pi^j\left(\tilde{\sigma}_{T+1}\right)}.$$

Hence,

$$\frac{u_i'\left(c^i\left(\sigma_0\right)\right)}{\beta_i u_i'\left(c^i\left(\omega^i_{T+1}\right)\right)\pi^i\left(\omega^i_{T+1}\right)} = \frac{u_j'\left(c^j\left(\sigma_0\right)\right)}{\beta_j\pi^j\left(\omega^i_{T+1}\right)\sum_{\tilde{\sigma}_{T+1}\in\omega^i_{T+1}} u_j'\left(c^j\left(\tilde{\sigma}_{T+1}\right)\right)\pi^j\left(\tilde{\sigma}_{T+1}\mid\omega^i_{T+1}\right)},$$

which reduces to:

$$\begin{aligned} \frac{u_i'\left(c^i\left(\omega^i_{T+1}\right)\right)}{\sum_{\tilde{\sigma}_{T+1}\in\omega^i_{T+1}} u_j'\left(c^j\left(\tilde{\sigma}_{T+1}\right)\right)\pi^j\left(\tilde{\sigma}_{T+1}\mid\omega^i_{T+1}\right)} &= \frac{\beta_j}{\beta_i}\frac{\pi^j\left(\omega^i_{T+1}\right)}{\pi^i\left(\omega^i_{T+1}\right)}\frac{u_i'\left(c^i\left(\sigma_0\right)\right)}{u_j'\left(c^j\left(\sigma_0\right)\right)} \\ &= \frac{u_i'\left(c^i\left(\sigma_0\right)\right)}{u_j'\left(c^j\left(\sigma_0\right)\right)}\prod_{t=1}^{T+1}\frac{\beta_j}{\beta_i}\frac{\pi^j\left(w^i_t\right)}{\pi^i\left(w^i_t\right)} \end{aligned}$$

and we obtain

$$\begin{aligned} \lim_{T\to\infty}\frac{1}{T+1}\ln\frac{u_i'\left(c^i\left(\omega^i_{T+1}\right)\right)}{\sum_{\tilde{\sigma}_{T+1}\in\omega^i_{T+1}} u_j'\left(c^j\left(\tilde{\sigma}_{T+1}\right)\right)\pi^j\left(\tilde{\sigma}_{T+1}\mid\omega^i_{T+1}\right)} &= \lim_{T\to\infty}\ln\frac{\beta_j}{\beta_i} + \lim_{T\to\infty}\frac{1}{T+1}\sum_{t=1}^{T+1}\ln\frac{\pi^j\left(w^i_t\right)}{\pi^i\left(w^i_t\right)} \\ &+ \lim_{T\to\infty}\frac{1}{T+1}\ln\frac{u_i'\left(c^i\left(\sigma_0\right)\right)}{u_j'\left(c^j\left(\sigma_0\right)\right)}. \end{aligned}$$

Since $u_i'\left(c^i\left(\sigma_0\right)\right)$ and $u_j'\left(c^j\left(\sigma_0\right)\right)$ are finite, the third term on the r.h.s. converges to 0, furthermore,

$$\begin{aligned}
&\lim_{T\to\infty}\frac{1}{T+1}\ln\frac{u_i'\left(c^i\left(\omega_{T+1}^i\right)\right)}{\sum_{\tilde{\sigma}_{T+1}\in\omega_{T+1}^i}u_j'\left(c^j\left(\tilde{\sigma}_{T+1}\right)\right)\pi^j\left(\tilde{\sigma}_{T+1}\mid\omega_{T+1}^i\right)}\\
=\;&\lim_{T\to\infty}\ln\frac{\beta_j}{\beta_i}+\lim_{T\to\infty}\frac{1}{T+1}\sum_{t=1}^{T+1}\left(\ln\pi^j\left(w_t^i\right)-\ln\pi\left(w_t^i\right)\right)+\\
&+\lim_{T\to\infty}\frac{1}{T+1}\sum_{t=1}^{T+1}\left(\ln\pi\left(w_t^i\right)-\ln\pi^i\left(w_t^i\right)\right).
\end{aligned}$$

Since $\ln\frac{\pi^j\left(w_t^i\right)}{\pi\left(w_t^i\right)}$ and $\ln\frac{\pi\left(w_t^i\right)}{\pi^j\left(w_t^i\right)}$ are i.i.d and are equal in expectations to the relative entropy of i's and j's beliefs w.r.t. the truth, we obtain that a.s.,

$$\begin{aligned}
&\lim_{T\to\infty}\frac{1}{T+1}\ln\frac{u_i'\left(c^i\left(\omega_{T+1}^i\right)\right)}{\sum_{\tilde{\sigma}_{T+1}\in\omega_{T+1}^i}u_j'\left(c^j\left(\tilde{\sigma}_{T+1}\right)\right)\pi^j\left(\tilde{\sigma}_{T+1}\mid\omega_{T+1}^i\right)}\\
=\;&\lim_{T\to\infty}\ln\frac{\beta_j}{\beta_i}+\left(\sum_{w^i\in W^i}\pi\left(w^i\right)\ln\frac{\pi\left(w^i\right)}{\pi^i\left(w^i\right)}-\sum_{w^i\in W^i}\pi\left(w^i\right)\ln\frac{\pi\left(w^i\right)}{\pi^j\left(w^i\right)}\right).
\end{aligned}$$

Proof of Proposition 6:

Using Lemma 27 and replacing the original state space Ω by the common partition, Ω, we obtain that for two such agents, i and j, with $\beta^i>\beta^j$:

$$\lim_{T\to\infty}\frac{1}{T+1}\ln\frac{u_i'\left(c^i\left(\omega_{T+1}\right)\right)}{u_j'\left(c^j\left(\omega_{T+1}\right)\right)}=\lim_{T\to\infty}\ln\frac{\beta_j}{\beta_i}<0$$

and hence $u_j'\left(c^j\left(\omega_{T+1}\right)\right)\to\infty$, or $c^j\left(\omega_{T+1}\right)\to0$.

When beliefs differ,

$$\lim_{T\to\infty}\frac{1}{T+1}\ln\frac{u_i'\left(c^i\left(\omega_{T+1}\right)\right)}{u_j'\left(c^j\left(\omega_{T+1}\right)\right)}=\lim_{T\to\infty}\ln\frac{\beta_j}{\beta_i}+\lim_{T\to\infty}\frac{1}{T+1}\sum_{t=1}^{T+1}\ln\frac{\pi^j\left(w_t\right)}{\pi^i\left(w_t\right)}$$

and both beliefs and the actual distribution over states of the world are i.i.d.,

$$\lim_{T\to\infty}\frac{1}{T+1}\ln\frac{u_i'\left(c^i\left(\omega_{T+1}\right)\right)}{u_j'\left(c^j\left(\omega_{T+1}\right)\right)}=\ln\frac{\beta_j}{\beta_i}+\left(\sum_{w\in W^i}\pi\left(w\right)\ln\frac{\pi\left(w\right)}{\pi^i\left(w\right)}-\sum_{w\in W^i}\pi\left(w\right)\ln\frac{\pi\left(w\right)}{\pi^j\left(w\right)}\right).$$

Hence, for equal discount factors, the agent whose beliefs w.r.t. the common partition are closer to the truth survives, while the other vanishes. When both discount factors and beliefs differ, we conclude that lower discount factors can be offset by having beliefs closer to the truth and vice versa. In particular, if

$$\ln\frac{\beta_j}{\beta_i}+\left(\sum_{w}\pi\left(w\right)\ln\frac{\pi\left(w\right)}{\pi^i\left(w\right)}-\sum_{w}\pi\left(w\right)\ln\frac{\pi\left(w\right)}{\pi^j\left(w\right)}\right)<0\,,$$

$u_j'\left(c^j\left(\omega_{T+1}\right)\right) \to \infty$, or $c^j\left(\omega_{T+1}\right) \to 0$ and j vanishes.

Proof of Propositions 7 and 9:

We first prove Proposition 9 and then use the result to prove Proposition 7. Suppose that agent i has the coarser partition. We can again use the proof of Lemma 27 by replacing the original state space by the finer of the two partitions, Ω^j. We conclude that

$$\lim_{T\to\infty} \frac{1}{T+1} \ln \frac{u_i'\left(c^i\left(\omega_{T+1}^i\right)\right)}{\left(\sum_{\tilde{\omega}_{T+1}^j \in \omega_{T+1}^i} u_j'\left(c^j\left(\tilde{\omega}_{T+1}^j\right)\right)\pi^j\left(\tilde{\omega}_{T+1}^j \mid \omega_{T+1}^i\right)\right)} \quad (8)$$

$$= \ln \frac{\beta_j}{\beta_i} + \left(\sum_{w^i \in W^i} \pi\left(w^i\right) \ln \frac{\pi\left(w^i\right)}{\pi^i\left(w^i\right)} - \sum_{w^i \in W^i} \pi\left(w^i\right) \ln \frac{\pi\left(w^i\right)}{\pi^j\left(w^i\right)}\right).$$

Since consumption is finite, and hence, $u_j'\left(c^j\left(\tilde{\omega}_{T+1}^j\right)\right)$ cannot become 0, and since the sum of the probabilities in the denominator is 1, if

$$\ln \frac{\beta_j}{\beta_i} + \left(\sum_{w^i \in W^i} \pi\left(w^i\right) \ln \frac{\pi\left(w^i\right)}{\pi^i\left(w^i\right)} - \sum_{w^i \in W^i} \pi\left(w^i\right) \ln \frac{\pi\left(w^i\right)}{\pi^j\left(w^i\right)}\right) > 0\,,$$

i vanishes a.s.

We now turn to proving Proposition 7. Note that the r.h.s. of (8) equals 0. Take a path σ_t and assume that, on this path only, the agent with the finest partition j survives, whereas all agents with coarser partitions, $i \neq j$ vanish. It follows that an agent $i \neq j$ also vanishes on the set of paths ω^i such that $\sigma \in \omega^i$. Hence, the numerator on the l.h.s. of (8) would converge to ∞, whereas the denominator remains finite and the l.h.s. would be strictly positive, a contradiction. Hence, at least one of the partially aware agents has to survive on σ.

We can rewrite condition (8) equivalently for any two agents whose partitions are nested. Hence, if an agent with a coarser partition, i, survives (and hence, the numerator is finite), then the denominator has to remain finite as well. So any agent j with finer partitions than agent i can only vanish on an event with probability 0 relative to their beliefs, or (since j's beliefs are correct) on a 0-probability event w.r.t. the truth. We thus conclude that the agent with the finest partition j survives a.s.. Now consider an agent i with a coarser partition than j. Since j survives a.s. w.r.t. the truth, and thus, w.r.t. his own beliefs, the denominator of (8) w.r.t. i and j is finite a.s.. It then follows that i also survives a.s..

Proof of Proposition 8:

At the equilibrium prices, i's and j's optimization problems are given by (1). Endowments, discount factors and utility functions coincide. Further, beliefs coincide on the common partition representing contingencies of which both are aware. Hence, the only difference between the two problems concerns the measurability requirements: c^j has to be measurable relative to $\left(\Omega^j; \mathcal{F}^j\right)$, whereas c^i has to be measurable relative to the finer $\left(\Omega^i; \mathcal{F}^i\right)$. Rewriting j's problem as:

$$\arg\max_{c^j}\left\{\begin{array}{c} u\left(c^j\left(\sigma_0\right)\right)+\sum_{t=1}^{\infty}\beta^t\sum_{\omega_t^i\in\Omega_t^i}\pi^i\left(\omega_t^i\right)u\left(c^j\left(\omega_t^i\right)\right) \\ \text{s.t. } \sum_{t\in\mathbb{N}}\sum_{\omega_t^i\in\Omega_t^i}\sum_{\sigma_t\in\omega_t^i}p\left(\sigma_t\right)c^j\left(\omega_t^i\right) \\ \leq\sum_{t\in\mathbb{N}}\sum_{\omega_t^i\in\Omega_t^i}\sum_{\sigma_t\in\omega_t^i}p\left(\sigma_t\right)e\left(\omega_t^i\right) \\ c^j\left(\omega_t^i\right)=c^j\left(\omega_t'^i\right)\ \text{ whenever } \omega_t^i,\,\omega_t'^i\in\omega_t^j \text{ for some } \omega_t^j \end{array}\right\}$$

shows that i and j can be viewed as maximizing the same utility function at the same equilibrium prices and at the same initial endowment, but i has a strictly larger choice set than j. Hence, $V_0^i\left(c^i\right)\geq V_0^j\left(c^j\right)$ obtains in equilibrium.

Proof of Proposition 11:

We will first show that if the unawareness w.r.t. partition $\Omega^{\tilde{k}}$ in the economy is relevant, at least one of agents $1\ldots\left(\tilde{k}-1\right)$ has to survive. In particular, assume that only agents with indices $k\geq\tilde{k}$ with i the minimal of these indices survive on a given path ω^i. According to Definition 10, there are s and $s'\in w^{\tilde{k}}\subseteq w^i$ and $\epsilon>0$ such that for any σ, $\sigma'\in\omega^{\tilde{k}}\subseteq\omega^i$,

$$\lim_{t\to\infty}\sup\left[e\left(\sigma_t;s\right)-e\left(\sigma_t';s'\right)\right]>\epsilon\ .$$

Since for every $k>i$, k's consumption is measurable w.r.t. ω^i and since the partitions are nested, we have that for every t, every $\sigma_t\in\omega_t^i$ and $s'\in w_{t+1}^i$,

$$\sum_{k\geq i}c^k\left(\omega_{t+1}^i\right)\leq e\left(\sigma_t;s'\right), \tag{9}$$

and hence, for every $\sigma\in\omega^i$, on which states s and s' occur infinitely often,

$$\begin{aligned}\lim_{T\to\infty}\sup\sum_{j<i}c^j\left(\sigma_T\right) &= \lim_{T\to\infty}\sup\left[e\left(\sigma_T\right)-\sum_{k\geq i}c^k\left(\sigma_T\right)\right] \\ &\geq \lim_{T\to\infty}\sup\left[e\left(\sigma_T\right)-\min e\left(\sigma_{T-1};s'\right)\right] \\ &\geq \lim_{T\to\infty}\sup\left[e\left(\sigma_{T-1};s\right)-e\left(\sigma_{T-1};s'\right)\right]>\epsilon.\end{aligned}$$

Since s occurs infinitely often on almost every path $\sigma\in\omega^i$, and since there is a finite number of agents in the economy, and since the unawareness of all agents with indices larger than $\tilde{k}-1$ is relevant in the limit, at least one of the agents $j\leq\tilde{k}-1$ must survive a.s. on ω^i.

Let $i\in\left\{1\ldots\left(\tilde{k}-1\right)\right\}$ survive a.s.. Consider the agents $j<i$.

Suppose first that $i=2$ and hence $j=1$. Since j's unawareness is irrelevant in the limit, the total endowment of the economy, as well as the consumption of all other agents than j are measurable w.r.t. i's partition in the limit. Hence, so must be j's consumption and

$$\lim_{T\to\infty}\sup\sum_{\tilde{\omega}_{T+1}^j\in\omega_{T+1}^i}u_j'\left(c^j\left(\tilde{\omega}_{T+1}^j\right)\right)\pi^j\left(\tilde{\omega}_{T+1}^j\mid\omega_{T+1}^i\right)=\lim_{T\to\infty}\sup u_j'\left(c^j\left(\omega_{T+1}^i\right)\right)\ .$$

It follows that j's consumption will converge to 0 a.s. whenever j's survival index is strictly smaller than that of i. Hence, j will survive a.s., whenever his beliefs and discount factor are identical to that of i and vanish a.s. if his survival index is lower than that of i.

We now proceed by induction. Suppose that we have shown for $i = \kappa$ that agent $j < i$ vanishes a.s. if his survival index is lower than that of i and survives a.s. only if his discount factor and beliefs are equal to those of i. Consider $i = \kappa + 1$.

Since, in the limit, the total consumption in the economy has to be measurable relative to i's partition and since agent 2's consumption has to be measurable relative to his partition, in the limit, agent 1's consumption has to be measurable relative to agent's 2 partition. Consider first the case in which agent 1's survival index is lower than that of agent 2. We can then use the argument from the proof of Proposition 6 to show that 1 vanishes a.s. relative to 2. Then we are back to the case where $i = \kappa$,. So, the rest of the agents survive a.s. if their survival indexes are equal to that of i and vanish a.s. otherwise.

Assume, therefore, that agent 1's survival index is equal to that of agent 2 and hence, their beliefs and discount factors are identical. The argument from the proof of Proposition 6 shows that 2 survives a.s. on an event if and only if 1 survives on this event. Hence, either 1 and 2 both vanish a.s., in which case we are back to the case $i = \kappa - 2$, or they both survive with positive probability. Since their discount factors and beliefs on the coarser partition of agent 2 are identical, $\beta_1 = \beta_2, .\pi^1\left(w^2\right) = \pi^2\left(w^2\right)$ for all $w^2 \in W^2$ and since, in the limit, the consumption of both 1 and 2 is measurable w.r.t. 2's awareness partition, in the limit, we can aggregate these two agents into one with a utility function, $u_{12} =: \alpha u_1 + (1-\alpha) u_2$ for some α. Furthermore, in the limit, the equilibrium consumption of this agent will be measurable w.r.t. to 3's partition. We can thus write (8) for the aggregate of 1 and 2, and 3 and repeat the argument from above. Proceeding iteratively, we can thus show that under the conditions of the proposition, 1 can only survive with positive probability if the discount factors and the beliefs of all the agents $1...i-1$ are identical. If this is the case, then all agents will survive a.s. on the same event on which 1 survives. Since agent i survives a.s., and since the total consumption of the agents $1...i-1$ has to be measurable relative to i's partition in the limit, we conclude from Proposition 6 that the aggregate agent $1...i-1$ survives a.s. But since the number of agents is finite, this implies that at least one of the agents survives a.s. Hence, so do all the others.

If $i = \tilde{k} - 1$, we conclude that among agents $\left\{1 \ldots \left(\tilde{k} - 1\right)\right\}$, only those with discount factors and beliefs equal to those of $\tilde{k} - 1$ survive. If, in contrast, there is a j such that $\left(\tilde{k} - 1\right) \geq j = i + 1$, let i' be the agent with the finest partition, whose beliefs and discount factor are equal to those of i. By the same argument as above, we can aggregate the agents i' through i in the limit to obtain an agent ii', whose limit equilibrium consumption is measurable relative to j's partition, since the unawareness of agents $i...i'$ is irrelevant in the limit. We thus obtain:

$$\lim_{T\to\infty}\frac{1}{T+1}\ln\frac{u_j'\left(c^j\left(\omega_{T+1}^j\right)\right)}{\sum_{\tilde{\omega}_{T+1}^i\in\omega_{T+1}^j}u_{ii}'\left(c^{ii'}\left(\tilde{\omega}_{T+1}^i\right)\right)\pi^i\left(\tilde{\omega}_{T+1}^i\mid\omega_{T+1}^j\right)}=\lim_{T\to\infty}\frac{1}{T+1}\ln\frac{u_j'\left(c^j\left(\omega_{T+1}^j\right)\right)}{u_{ii}'\left(c^{ii'}\left(\tilde{\omega}_{T+1}^i\right)\right)}\geq 0\ .$$

Since all agents ii' survive a.s., it follows that j's beliefs and discount factor are identical to those of i (and all $i...i'$). (Otherwise, ii' would vanish a.s. relative to j in contradiction to the assumption that agent i survives a.s.). Replacing i by j, and repeating the argument iteratively, we conclude that if agent i survives a.s., so do all agents $i+1...\tilde{k}-1$. Furthermore, the beliefs and discount factor of agents $i+1...\tilde{k}-1$ have to be equal to those of i.

We thus conclude that under the conditions stated in the proposition, among the agents $\left\{1\ldots\tilde{k}-1\right\}$, agent k survives a.s. if

$$\left\{\ln\frac{\beta_{\tilde{k}-1}}{\beta_k}-\sum_{w^{\tilde{k}-1}}\pi\left(w^{\tilde{k}-1}\right)\ln\frac{\pi^k\left(w^{\tilde{k}-1}\right)}{\pi^{\tilde{k}-1}\left(w^{\tilde{k}-1}\right)}\right\}=0$$

and vanishes a.s. otherwise. In particular, agent $\left(\tilde{k}-1\right)$ survives a.s..

Now consider agent $j>\left(\tilde{k}-1\right)$. Suppose that all agents $i...\tilde{k}-1$ survive a.s. and thus have identical discount factors and identical beliefs. Since in the limit, their total equilibrium consumption will be measurable relative to $\tilde{k}-1$'s awareness partition, in the limit, we can aggregate them into an agent $i\left(\tilde{k}-1\right)$. For $j>\tilde{k}-1$, we then obtain

$$\begin{aligned}&\lim_{T\to\infty}\frac{1}{T+1}\ln\frac{u_j'\left(c^j\left(\omega_{T+1}^j\right)\right)}{\sum_{\tilde{\omega}_{T+1}^{\tilde{k}-1}\in\omega_{T+1}^j}u_{i\left(\tilde{k}-1\right)}'c^{i\left(\tilde{k}-1\right)}\left(\tilde{\omega}_{T+1}^{\tilde{k}-1}\right)\Pi_{t=1}^T\pi^{\tilde{k}-1}\left(\tilde{w}_{T+1}^{\tilde{k}-1}\mid w_{T+1}^j\right)} \qquad (10)\\ =\;&\lim_{T\to\infty}\ln\frac{\beta_{\tilde{k}-1}}{\beta_j}+\left(\sum_{w^j\in W^j}\pi\left(w^j\right)\ln\frac{\pi\left(w^j\right)}{\pi^j\left(w^j\right)}-\sum_{w^j\in W^j}\pi\left(w^j\right)\ln\frac{\pi\left(w^j\right)}{\pi^{\tilde{k}-1}\left(w^j\right)}\right)\ .\end{aligned}$$

Since j's survival index is greater or equal to that of $\tilde{k}-1$, the limit on the r.h.s. is non-positive. Since the aggregate agent $i\left(\tilde{k}-1\right)$ survives a.s., and furthermore, since his consumption is strictly positive and bounded away from 0 in the limit on at least one state s, we have that on all paths, on which s occurs infinitely often,

$$\lim_{T\to\infty}\inf\sum_{\tilde{\omega}_{T+1}^{\tilde{k}-1}\in\omega_{T+1}^j}u_{i\left(\tilde{k}-1\right)}'c^{i\left(\tilde{k}-1\right)}\left(\tilde{\omega}_{T+1}^{\tilde{k}-1}\right)\Pi_{t=1}^T\pi^{\tilde{k}-1}\left(\tilde{w}_{T+1}^{\tilde{k}-1}\mid w_{T+1}^j\right)<\infty.$$

Hence, if j were to vanish, the l.h.s. would remain strictly positive, whereas the r.h.s. of (10) is non-positive, a contradiction. Hence, every $j>\tilde{k}-1$ survives a.s..

Proof of Proposition 13:

Note that for all agents other than j, $c^i(\sigma_t; s) = c^i(\sigma_t; s')$ has to hold in equilibrium. Since condition (4) is satisfied, agent j's consumption on state s is bounded below by ϵ in the limit. Since state s occurs infinitely often on almost every path, we conclude that j survives a.s..

Whenever the economy has non-nested partitions as in Definition 12, the same argument applies to every agent $j \in I$ and the statement of the Proposition obtains.

Proof of Proposition 14:

Consider a set of paths $\bar{\Omega}$ which has a strictly positive probability and on which the fully aware agent j vanishes a.s.. We know from the previous result that, on this set of paths, all partially aware agents survive a.s.. Hence, take the minimal set $\bar{\Omega}'$ such that $\bar{\Omega}' \supseteq \bar{\Omega}$ and $\bar{\Omega}' \in F^i$. The set $\bar{\Omega}'$ also has a strictly positive probability and since i survives on $\bar{\Omega}$, he also survives on $\bar{\Omega}'$. It follows that for any path $\omega^i \in \bar{\Omega}'$, the numerator of (8) remains finite and hence, the denominator is also finite. Hence, by the argument in the proof of Proposition 7, the fully aware agent survives a.s. on every such ω^i, a contradiction. We conclude thus that agent j survives a.s..

Since j survives a.s., we know from the argument used in the proof of Proposition 7 that any agent with a coarser awareness partition than j, correct beliefs and an identical discount factor also survives a.s..

Proof of Proposition 19:

The proof is by construction: we will use the endowments $e^1 ... e^n$ to construct the asset endowments $\bar{a}^1 ... \bar{a}^n$, which will imply the same equilibrium consumption. To do so, first construct an effectively complete asset structure $\mathcal{A}$ using the method described in the paragraph after Definition 16. Set the initial endowments to: $\bar{a}^j\left(A_{\omega_t^j}\right) = e^j\left(\omega_t^j\right)$ for all $j \in I$, $\bar{a}^j\left(A_{\omega_t^k}\right) = 0$ if $\omega_t^k \notin \Omega^j$. In the following, we will use $a^j\left(\omega_t^k\right)$ as a short hand for $a^j\left(A_{\omega_t^k}\right)$.

For every agent $j \neq i$, set $a^j\left(\omega_t^j\right) = c^j\left(\omega_t^j\right)$ for all $\omega_t^j \in \Omega^j$ and $a^j\left(\omega_t^k\right) = 0$ for all $\omega_t^k \notin \Omega_t^j$. Hence, for each agent $j \neq i$, his asset holdings are measurable relative to his awareness partition and generate exactly his equilibrium consumption. For agent i and $k \in I \backslash \{i\}$ and $\omega_t^k \in \Omega^k$, let

$$a^i\left(\omega_t^k\right) = \sum_{\left\{j \mid \omega_t^k \in \Omega^j,\ j \neq i\right\}} \left[e^j\left(\omega_t^k\right) - c^j\left(\omega_t^k\right)\right] + e^i\left(\omega_t^k\right).$$

For $\omega_t^i \in \Omega^i \backslash \left(\cup_{k \in I \backslash \{i\}} \Omega^k\right)$, set $a^i\left(\omega_t^i\right) = \bar{a}^i\left(\omega_t^i\right)$. In the latter case, the asset market for $A_{\omega_t^i}$ trivially clears, since only i is aware of asset $A_{\omega_t^i}$.

For $\omega_t^k \in \Omega^k$ for some $k \neq i$, by construction,

$$\begin{aligned} a^i\left(\omega_t^k\right) &= \bar{a}^i\left(\omega_t^k\right) + \sum_{\left\{j \mid \omega_t^k \in \Omega^j,\ j \neq i\right\}} \left[\bar{a}^j\left(\omega_t^k\right) - a^j\left(\omega_t^k\right)\right] \\ &= \bar{a}^i\left(\omega_t^k\right) + \sum_{j \in I,\ j \neq i} \left[\bar{a}^j\left(\omega_t^k\right) - a^j\left(\omega_t^k\right)\right], \end{aligned}$$

or

$$\sum_{j\in I} a^j\left(\omega_t^k\right) = \sum_{j\in I} \bar{a}^j\left(\omega_t^k\right)$$

and asset markets clear for all assets $A_{\omega_t^k}$ with $\omega_t^k \in \Omega^k$ for some $k \neq i$.

Since Ω^i is finer than any of the Ω^j for $j \neq i$, a^i is measurable relative to i's awareness partition. Furthermore, for every $\omega_t^i \in \Omega^i$ and every $j \neq i$, there is a corresponding element of j's awareness partition, $\omega_t^j\left(\omega_t^i\right)$ such that $\omega_t^j\left(\omega_t^i\right) \in \Omega^j$ and $\omega_t^i \subseteq \omega_t^j\left(\omega_t^i\right)$. We then obtain that the payoff of i's portfolio a^i on the event ω_t^i is given by:

$$\begin{aligned}\sum_{\left\{\omega_t^k \mid k\in I\setminus\{i\},\ \omega_t^k\in\Omega^k,\ \omega_t^k\supseteq\omega_t^i\right\}} a^i\left(\omega_t^k\right) &= \sum_{j\in I,\ j\neq i}\left[e^j\left(\omega_t^j\left(\omega_t^i\right)\right) - c^j\left(\omega_t^j\left(\omega_t^i\right)\right)\right] + e^i\left(\omega_t^i\right) = \\ &= \sum_{j\in I,\ j\neq i}\left[e^j\left(\omega_t^i\right) - c^j\left(\omega_t^i\right)\right] + e^i\left(\omega_t^i\right) = c^i\left(\omega_t^i\right)\end{aligned}$$

and is, therefore equal to i's equilibrium consumption on ω_t^i.

Our construction thus shows that we can replicate the equilibrium consumption streams of all agents by choosing portfolios which are measurable relative to the agents' awareness partitions, generate the equilibrium consumption streams and clear the asset markets. Let $\left(p\left(\sigma_t\right)\right)_{\sigma_t\in\Omega}$ be the equilibrium price sequence. For each asset $A_{\omega_t^k}$, let the price of this asset be given by

$$q_{A_{\omega_t^k}} = \sum_{\sigma_t\in\omega_t^k} p\left(\sigma_t\right).$$

Then the so defined equilibrium portfolios indeed maximize the agents' utilities when the asset prices are given by $\left(q_A\right)_{A\in\mathcal{A}}$, which completes the proof of the proposition.

Proof of Proposition 20:

As in the proof of Proposition 19, let $\bar{a}^k\left(\omega_t^k\right) = e^k\left(\omega_t^k\right)$ for $k \in \{i;j\}$ and $\omega_t^k \in \Omega^k$ be the initial endowment of the agents with assets. Let Ω^{ij} be the finest coarsening of the two partitions Ω^i and Ω^j and let generalized Arrow securities for all events $\omega_t^{ij} \in \Omega^{ij}\setminus\left(\Omega^i \cup \Omega^j\right)$ be available in 0 supply.

Note that in this economy, the only trades that can occur in equilibrium have to be measurable relative to Ω^{ij}. Hence, if $c^i\left(\omega_t^i\right) \neq e^i\left(\omega_t^i\right)$ for some $\omega_t^i \in \Omega^i$, then we must have that

$$c^i\left(\sigma_t\right) - e^i\left(\sigma_t\right) = const \text{ for all } \sigma_t \in \omega_t^{ij},$$

where ω_t^{ij} is the minimal element in Ω^{ij} such that $\omega_t^{ij} \supseteq \omega_t^i$. Hence, consider the following asset holdings:

$$\begin{aligned}\text{if } \omega_t^k &\notin \Omega^{ij},\ a^k\left(\omega_t^k\right) = \bar{a}^k\left(\omega_t^k\right) = e^k\left(\omega_t^k\right); \\ \text{if } \omega_t^k &\in \Omega^{ij},\ \omega_t^k \notin \Omega_t^k,\ a^k\left(\omega_t^k\right) = c^k\left(\tilde{\omega}_t^k\right) - e^k\left(\tilde{\omega}_t^k\right) \text{ for some, and thus, for all } \tilde{\omega}_t^k \subset \omega_t^k,\ \tilde{\omega}_t^k \in \Omega^k; \\ \text{if } \omega_t^k &\in \Omega^{ij} \cap \Omega_t^k,\ a^k\left(\omega_t^k\right) = c^k\left(\omega_t^k\right).\end{aligned}$$

Given these asset holdings, for every $\omega_t^k \in \Omega^k \backslash \Omega^{ij}$ such that $\omega_t^k \in \tilde{\omega}_t^k$ for $\tilde{\omega}_t^k \in \Omega^{ij}$, the consumption of agent k on ω_t^k is given by:

$$a^k\left(\tilde{\omega}_t^k\right) + a^k\left(\omega_t^k\right) = e^k\left(\omega_t^k\right) + c^k\left(\omega_t^k\right) - e^k\left(\omega_t^k\right) = c^k\left(\omega_t^k\right),$$

and hence, for each $\omega_t^k \in \Omega^k$, k's consumption exactly corresponds to k's equilibrium consumption at ω_t^k. Furthermore, the asset holdings clear the market, since on $\omega_t^k \notin \Omega^{ij}$ for i, j,

$$a^i\left(\omega_t^k\right) + a^j\left(\omega_t^k\right) = \bar{a}^i\left(\omega_t^k\right) + \bar{a}^j\left(\omega_t^k\right),$$

on $\omega_t^k \in \Omega^{ij}$ such that $\omega_t^k \in \Omega_t^k$ for $k \in \{i; j\}$,

$$a^i\left(\omega_t^k\right) + a^j\left(\omega_t^k\right) = c^i\left(\omega_t^k\right) + c^j\left(\omega_t^k\right) = e^i\left(\omega_t^k\right) + e^j\left(\omega_t^k\right) = \bar{a}^i\left(\omega_t^k\right) + \bar{a}^j\left(\omega_t^k\right),$$

on $\omega_t^k \in \Omega^{ij}$ such that $\omega_t^k \notin \Omega_t^k$ for $k \in \{i; j\}$,

$$\begin{aligned} a^i\left(\omega_t^k\right) + a^j\left(\omega_t^k\right) &= c^i\left(\tilde{\omega}_t^i\right) + c^j\left(\tilde{\omega}_t^j\right) - e^i\left(\tilde{\omega}_t^i\right) - e^j\left(\tilde{\omega}_t^j\right) \\ &= c^i\left(\sigma_t\right) + c^j\left(\sigma_t\right) - e^i\left(\sigma_t\right) - e^j\left(\sigma_t\right) = 0 = \bar{a}^i\left(\omega_t^k\right) + \bar{a}^j\left(\omega_t^k\right) \end{aligned}$$

for some, and thus for all $\tilde{\omega}_t^i \subset \omega_t^k$, $\tilde{\omega}_t^i \in \Omega^i$ and $\tilde{\omega}_t^j \subset \omega_t^k$, $\tilde{\omega}_t^j \in \Omega^j$, $\sigma_t \in \omega_t^k$, and finally, for $\omega_t^k \in \Omega^{ij}$ such that $\omega_t^k \notin \Omega_t^i$, $\tilde{\omega}_t^i \subset \omega_t^k$, $\tilde{\omega}_t^i \in \Omega^i$, $\omega_t^k \in \Omega_t^j$,

$$\begin{aligned} a^i\left(\omega_t^k\right) + a^j\left(\omega_t^k\right) &= c^i\left(\tilde{\omega}_t^i\right) - e^i\left(\tilde{\omega}_t^i\right) + c^j\left(\omega_t^j\right) = c^i\left(\sigma_t\right) - e^i\left(\sigma_t\right) + c^j\left(\sigma_t\right) = e^j\left(\sigma_t\right) \\ &= e^j\left(\omega_t^k\right) = \bar{a}^j\left(\omega_t^k\right) = \bar{a}^i\left(\omega_t^k\right) + \bar{a}^j\left(\omega_t^k\right) \end{aligned}$$

for all $\sigma_t \in \omega_t^k$. As long as the asset prices satisfy the arbitrage condition

$$q_{A_{\omega_t^k}} = \sum_{\sigma_t \in \omega_t^k} p\left(\sigma_t\right)$$

these portfolio holdings will also maximize the agents' expected utility given the budget constraint. Finally, the selected portfolios contain only assets whose payoffs are measurable relative to the respective awareness partitions.

Proof of Proposition 23:

The existence of such an equilibrium follows easily from Bewley's (1972) theorem. If all agents are aware of $\sigma_{t^*}^*$, then all agents assign 0-probability to all $\sigma_t \notin \Omega_{\sigma_{t^*}^*}$ and we can directly apply the result of Proposition 2. If not all agents are aware of $\sigma_{t^*}^*$, then the set of contingencies in this economy includes all $\sigma_t \in \omega_t^{*i}$ for all $\omega_t^{*i} \in \Omega_{\sigma_{t^*}^*}^{*i}$ for some $i \in I$. Obviously, given $\sigma_{t^*}^*$, many of the paths have an objective probability of 0 and are assigned 0-probability by agents who are aware of $\sigma_{t^*}^*$. In contrast, agents who are not aware of $\sigma_{t^*}^*$ assign (mistakenly) strictly positive probability to impossible events. Hence, in equilibrium, there will be potentially trade over 0-probability contingencies: agents who are aware of $\sigma_{t^*}^*$ will want to sell consumption contingent on $\sigma_t \notin \Omega_{\sigma_{t^*}^*}$, whereas agents unaware of $\sigma_{t^*}^*$ would like to buy it. Non-negativity constraints on consumption ensure that such trades remain finite. Given Assumption 3, all agents will wish to assign strictly positive consumption to nodes $\sigma_t^* \in \Omega_{\sigma_{t^*}^*}$ such that as long as $p(\sigma_t^*) \in (0;\infty)$. However, if $p(\sigma_t) \in (0;\infty)$, only an agent j for whom $\sigma_t \in \omega_t^{*j} \backslash \Omega_{\sigma_{t^*}^*}$ will wish to assign strictly positive consumption to such a node, whereas all agents aware of $\sigma_{t^*}^*$ will want to consume 0 at σ_t. We will now show that this cannot constitute an equilibrium allocation and hence, $p(\sigma_t) = 0$ has to hold, whenever $\sigma_t^* \notin \Omega_{\sigma_{t^*}^*}$.

Take $\sigma_t \notin \Omega_{\sigma_{t^*}^*}$ such that there is an $l \in I$ and a $\omega_t^{*l} \in \Omega_{\sigma_{t^*}^*}^{*l}$ such that $\sigma_t \in \omega_t^{*l}$. Let L denote the set of all such l. Suppose that $c^{*k}(\sigma_t) = 0$ for all $k \notin L$. Then, $\sum_{l \in L} c^{*l}(\sigma_t) = e(\sigma_t)$. Let $l' \in L$ be the agent in L with the finest partition. Then there is also a node $\sigma_t^* \in \Omega_{\sigma_{t^*}^*}$ such that σ_t and $\sigma_t^* \in \omega_t^{*l'}$ and hence, $\sigma_t^* \in \omega_t^{*l}$ for all $l \in L$. By Assumption 4, we can choose σ_t^* so that $e(\sigma_t^*) \le e(\sigma_t)$. Measurability of consumption implies that $\sum_{l \in L} c^{*l}(\sigma_t) = \sum_{l \in L} c^{*l}(\sigma_t^*)$. However, we know that $c^{*k}(\sigma_t) = 0 < c^{*k}(\sigma_t^*)$ for all k aware of $\sigma_{t^*}^*$ whenever $p(\sigma_t) > 0$, hence, this cannot be an equilibrium allocation. We conclude that $p(\sigma_t) = 0$ for all $\sigma_t \notin \Omega_{\sigma_{t^*}^*}$. Hence, the f.o.c.s characterizing the equilibrium upon retrading in such an economy will coincide with the respective f.o.c.s in an economy with an initial node σ_0 replaced by $\sigma_{t^*}^*$. Hence, we can use the results from Sections 3 and 4 to characterize the equilibrium and survival.

Proof of Lemma 25:

The proof of the Lemma is obvious and therefore omitted.

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