

# On the Weights of Sovereign Nations

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## Introduction

- ▶ International unions: group of countries agree to make collective decisions on specific areas.
- ▶ Sovereign countries  $\Rightarrow$  participation is voluntary.
- ▶ **Ex-ante**, choice to cooperate reflects a trade off between:
  - ▶ increased efficiency: coordination externalities, economies of scale, increased bargaining power, etc
  - ▶ loss of decision power: countries must sometimes implement unfavorable decisions.
- ▶ **Ex-post**, choice to respect unfavorable collective decisions reflects a trade off between:
  - ▶ Loss of implementing an unfavorable proposal today.
  - ▶ Gain of maintaining cooperation in future.
- ▶ Voting rule crucial in explaining whether countries cooperate or not.
- ▶ **Second best** constitutional design: Optimal voting rule under voluntary participation?

## Historical examples: voluntary participation matters

### US Constitution

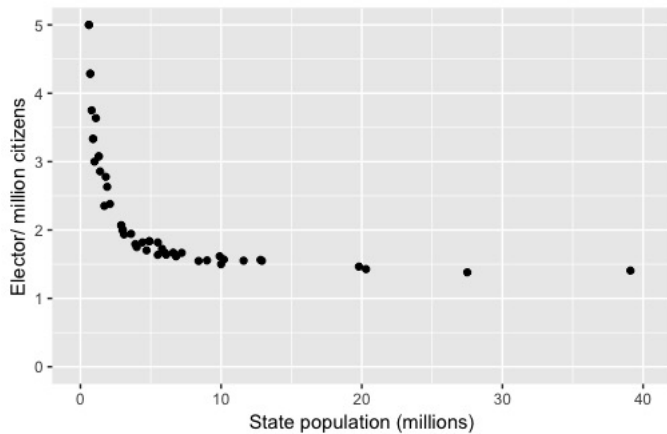
- ▶ US Constitutional Convention of 1787: most contentious issue revolved around the composition of the legislature.
- ▶ Larger states argued in favor of proportional representation.
- ▶ Smaller states argued instead in favor of equal representation, threatening to leave if the proportional solution was implemented.

*“The small ones would find some foreign ally of more honor and good faith, who will take them by the hand and do them justice.”*

Gunning Bedford Jr., representative for Delaware, 1787.

- ▶ Resolved by the *Connecticut Compromise*: bicameral legislature with proportional representation in the House and equal representation in the Senate.

## The US Electoral College (2017)



## Historical examples: voluntary participation matters

### UN Security Council

- ▶ A resolution at the Security Council is accepted if approved by at least 9 countries (over 15), not vetoed by any of the 5 permanent members.
- ▶ Veto power often criticized for severely reducing the efficiency of the UN.
- ▶ However, when the Charter of the UN was ratified in San Francisco in 1945, the issue was made crystal clear by the leaders of the Big Five: it was either the Charter with the veto or no Charter at all.

*“You may if you wish go home from this Conference and say that you have defeated the veto. But what will be your answer when you are asked: Where is the Charter?”*

U.S. Senator Tom Connally at the 1945 San Francisco Conference.

# Introduction

## The model

### Sketch of the Model:

- ▶ A fixed group of countries must decide whether to cooperate on a specific area, or remain sovereign.
- ▶ If cooperation is agreed, countries take repeated (independent) collective decisions according to a predetermined voting rule.
- ▶ If cooperation is rejected, each country remains sovereign and makes its own independent decisions.
- ▶ Crucially, cooperation is agreed ex-ante, before countries learn their preferences over future decisions.
- ▶ Decision to cooperate based on expected utility from any such collective decision.
- ▶ If decisions enforceable, then countries must abide by the collective decision. If not, they are free to go against the collective decision.

## Introduction

### Our conclusions:

- ▶ Inducing voluntary participation may require giving more voting power to countries relative to what efficiency recommends.
- ▶ Participation constraints stronger under non-enforceable decisions, may require granting veto power to certain countries.
- ▶ Application to a model of **apportionment**: countries differ only in population size, binary utilities, ex-ante identical preferences.
- ▶ Under **enforceable** decisions: Countries must receive weights proportional to their populations, except for the smallest ones, which must all be weighted equally.
- ▶ Under **non-enforceable** decisions: optimal voting rule never recommends veto power for a subset of countries; either the rule must be a weighted majority rule with no veto or it must be the unanimity rule.

# Model

## Voting rules

Fixed group of countries:  $N = \{1, \dots, n\}$ .

Binary decisions:

- ▶ 0 (status quo) or 1 (reform)
- ▶ country  $i$  votes  $m_i \in \{0, 1\}$
- ▶ Voting rule:  $v : \{0, 1\}^N \rightarrow [0, 1]$ .

$v$  is **weighted** if  $\exists[\mathbf{w}; t]$  such that

$$\begin{cases} \sum_{i, m_i=1} w_i > t \sum_{i \in N} w_i & \Rightarrow v(\mathbf{m}) = 1 \\ \sum_{i, m_i=1} w_i < t \sum_{i \in N} w_i & \Rightarrow v(\mathbf{m}) = 0 \end{cases}$$

$i$  has **veto power** if  $m_i = 0 \Rightarrow v(\mathbf{m}) = 0$ . Veto countries:  $VE(v)$ .



# Model

## Decision game

Model extends Barbera and Jackson (2006) with pre and post decision stages.

**First stage:** Countries decide whether to cooperate ( $d_i = 1$ ) or remain sovereign ( $d_i = 0$ ).

- ▶  $d_i = 1$  for all  $i \in N$ : cooperation.
- ▶  $d_i = 0$  for some  $i \in N$ : no cooperation, every country  $i \in N$  remains sovereign and gets its **stand-alone utility**  $U_i^\emptyset$

**Second stage:** countries learn their utility ( $u_i \in \mathbb{R}$ ) from a proposed reform.

- ▶  $u_i$  is drawn from a **distribution**  $\mu_i$ , independently across countries.
- ▶ country  $i$  favors the reform if  $u_i > 0$ , the status quo if  $u_i < 0$ .

# Model

## Decision game

**Third stage:** countries vote in favor ( $m_i = 1$ ) or against ( $m_i = 0$ ) the reform.

- ▶ Collective decision  $v(\mathbf{m}) \in \{0, 1\}$  taken according to predetermined voting rule  $v$ .

**Fourth stage:** countries implement the reform ( $a_i = 1$ ) or not ( $a_i = 0$ ):

- ▶ **enforceable** case:  $a_i = v(\mathbf{m}) \quad \forall i \in N$ .
- ▶ **non-enforceable** case: the choice of action  $a_i$  is free.
- ▶ The reform is of a “pure collective action” form: effective (yields  $u_i$ ) only if applied by all countries.
- ▶ If the proposal is not implemented effectively, the status quo prevails and all countries get utility 0.

# Model

## Decision game

Focus on the **cooperative profile** of the game:

- ▶ Countries cooperate ( $d_i = 1$ ),
- ▶ Vote truthfully ( $m_i = \mathbb{1}_{u_i > 0}$ ),
- ▶ Respect the collective decision ( $a_i = v(\mathbf{m})$ ).

Associated expected utility:  $U_i(v) = \mathbb{E}[v((\mathbb{1}_{u_j > 0})_{j \in N})u_i]$

**Enforcable decisions:** Perfect Bayesian Equilibrium of the decision game.

**Non-Enforcable decisions:** Public Perfect Equilibrium of the associated discounted infinitely repeated game.

# Model

## Illustration

A group of 5 countries must decide, repeatedly, whether to impose embargoes on tax havens (effective only if approved by all countries)

Country 1 is generally **unfavorable**:

- ▶ with proba  $1/3$ ,  $u_1 = 1 > 0$ .
- ▶ with proba  $2/3$ ,  $u_1 = -2 < 0$ .

Countries  $i = 2, 3, 4, 5$  are generally **favorable**:

- ▶ with proba  $2/3$ ,  $u_i = 2 > 0$ .
- ▶ with proba  $1/3$ ,  $u_i = -1 < 0$ .

Utilities are drawn independently across countries and decisions.

### Benchmark 1: Sovereignty (no cooperation)

- ▶ Independent national decisions.
- ▶  $\mathbb{P}(\textit{embargo}) = 2^4/3^5 \approx 0.06$ .
- ▶ Utilities:  $U_1^\emptyset = 16$ ,  $U_{2,3,4,5}^\emptyset = 32$ .
- ▶ Welfare:  $W^\emptyset = 144$ .

### Benchmark 2: Cooperation under the efficient voting rule $\mathbf{v}^e$ (max $W$ )

- ▶ Simple majority rule: weights  $w_i^e = 1 \forall i$ , threshold  $t^e = 1/2$ .
- ▶  $\mathbb{P}(\textit{embargo}) = 168/2^5 \approx 0.69$ .
- ▶ Utilities:  $U_1^e = -120$ ,  $U_{2,3,4,5}^e = 228$ .
- ▶ Welfare:  $W^e = 792 \gg W^\emptyset = 144$ .

$\Rightarrow$  Ex-ante, country 1 not willing to cooperate:  $U_1^e = -120 < U_1^\emptyset = 16$ .

**Second-best approach:** Enforceable collective decisions.

- ▶ Cooperative profile PBE if  $U_i \geq U_i^\emptyset$  for all  $i \in N$ .
- ▶ Constrained Maximization Program:  $\max W$ , s.t.  $U_i(v) \geq U_i^\emptyset \quad \forall i \in N$

**Optimal rule  $\mathbf{v}^*$ :** weighted majority

- ▶ Weights:  $w_1^* = 3 > w_1^e$ ,  $w_{2,3,4,5}^* = 1 = w_{2,3,4,5}^e$ .
- ▶ Threshold  $t^* = 1/2$ .
- ▶ Utilities:  $U_1^* = 16 = U_1^\emptyset$ ,  $U_{2,3,4,5}^* = 146$ .
- ▶ Social welfare is  $W^* = 600$ .

**Second-best approach:** Non-Enforceable collective decisions.

- ▶ Countries may now choose whether to implement or not voted reforms ex-post.
- ▶ Incentives to comply comes from repeated interaction.
- ▶ Voting rule self enforcing if cooperative profile is a PPE of the associated discounted infinitely repeated game (here for  $\delta = 5/6$ ).

**Optimal self-enforcing rule  $\mathbf{v}^{**}$ :** weighted supermajority.

- ▶ Weights  $w_1^{**} = 3$ ,  $w_{2,3,4,5}^{**} = 1$ .
- ▶ Threshold  $t^{**} = 2/3 \Rightarrow$  country 1 has **veto power** ( $4/7 < 2/3$ ).
- ▶ Utilities:  $U_1^{**} = 72$ ,  $U_{2,3,4,5}^{**} = 84$ .
- ▶ Social welfare is  $W^{**} = 408$ .

## Benchmark: efficient voting rule.

(Barberà and Jackson, 2006)

- ▶ Welfare:  $W(v) = \sum_{i \in N} U_i(v) \Rightarrow$  efficient voting rule:  $\max_v W(v)$
- ▶ Country  $i$ 's stake:  $w_i^e = w_i^+ + w_i^-$ , where:

$$w_i^+ = \mathbb{E}[u_i | u_i > 0], \quad w_i^- = -\mathbb{E}[u_i | u_i < 0]$$

- ▶ Country  $i$ 's favored threshold:  $t_i^e = \frac{w_i^-}{w_i^+ + w_i^-}$ .

**Theorem** (Barberà and Jackson, 2006; Azrieli and Kim, 2014)

Any efficient voting rule  $v^e$  is a weighted majority rule. It is represented by  $[\mathbf{w}^e; t^e]$ , where the threshold  $t^e$  is defined by:

$$t^e = \frac{\sum_{i \in N} w_i^e t_i^e}{\sum_{i \in N} w_i^e},$$

- ▶ Efficient weight  $w_i^e$  reflects country  $i$ 's stake in the collective decision.



# Enforceable decisions

## Participation constraints

We first consider the decision game with **enforceable decisions**.

- ▶ The cooperative profile is a perfect Bayesian equilibrium of the decision game with enforceable decisions if and only if:  $U_i(v) \geq U_i^\emptyset \quad \forall i \in N$ .
- ▶ A rule  $v^*$  is **optimal** if it solves:

$$\left| \begin{array}{l} \max_v \quad W(v) \\ \text{s.t.} \quad \forall i \in N, \quad U_i(v) \geq U_i^\emptyset \end{array} \right.$$

## Theorem

There exists a system of weights  $[\mathbf{w}^*; t^*]$  such that any optimal rule  $v^*$  is represented by  $[\mathbf{w}^*; t^*]$  and such that:

$$\forall i \in N, \quad \left\{ \begin{array}{l} U_i(v^*) = U_i^\emptyset \quad \Rightarrow \quad w_i^* \geq w_i^e \\ U_i(v^*) > U_i^\emptyset \quad \Rightarrow \quad w_i^* = w_i^e \end{array} \right. \quad \text{and} \quad t^* = \frac{\sum_{i \in N} w_i^* t_i^e}{\sum_{i \in N} w_i^*}$$

# Enforceable decisions

## Optimal voting rules

- ▶ The optimal rule is weighted and essentially unique.
- ▶ Overweighted countries have a binding participation constraint (as for country 1 in the illustration).
- ▶ Contrary to efficient weights, optimal weights depend on the whole distribution of preferences  $(\mu_i)_{i \in N}$  and stand-alone utilities  $(U_i^\emptyset)_{i \in N}$ .
- ▶ A country may be overweighted at the optimum if it gains relatively little from cooperation or if it often disagrees with the (efficient) collective decision.
- ▶ The level of heterogeneity across countries, both in stakes and preferences, thus plays a crucial role in determining the optimal rule.
- ▶ In general, difficult to a priori identify overweighted countries. Possible under additional restrictions (e.g. in the apportionment model).

# Non-enforceable decisions

## Infinitely repeated game

Discounted infinitely repeated game with  $\delta \in (0, 1)$

At each step  $T$ , each country  $i$ :

- ▶ Chooses whether to cooperate:  $d_i^T \in \{0, 1\}$ .
- ▶ Privately observes  $u_i^T$  (draws are independent across steps).
- ▶ Votes in favor or against the reform at stage  $T$ :  $m_i^T \in \{0, 1\}$ .
- ▶ Observes the collective decision  $v(\mathbf{m}^T)$ .
- ▶ Implements or not the reform:  $a_i^T \in \{0, 1\}$ .

## Definition

A voting rule  $v$  is **self-enforcing** if there exists a *perfect public equilibrium* of the  $\delta$ -discounted infinitely repeated game such that the cooperating profile is played at each stage of the game on the equilibrium path.

# Non-enforceable decisions

## Self-enforcing rules

### Theorem

For any optimal self-enforcing rule  $v^{**}$ , there exists a system of weights  $[\mathbf{w}^{**}; t^{**}]$  representing  $v^{**}$  and such that:

$$\left\{ \begin{array}{l} U_i(v^{**}) < U_i^\emptyset + \frac{1-\delta}{\delta} w_i^D \Rightarrow w_i^{**} > w_i^e \text{ and } i \in VE(v^{**}) \\ U_i(v^{**}) = U_i^\emptyset + \frac{1-\delta}{\delta} w_i^D \Rightarrow w_i^{**} \geq w_i^e \\ U_i(v^{**}) > U_i^\emptyset + \frac{1-\delta}{\delta} w_i^D \Rightarrow w_i^{**} = w_i^e \text{ and } i \notin VE(v^{**}) \end{array} \right.$$

and

$$t^{**} \geq \frac{\sum_{i \in N} w_i^{**} t_i^e}{\sum_{i \in N} w_i^{**}}$$

where  $w_i^D$  is country  $i$ 's maximal disutility from the reform:

$$w_i^D = \max \{ w \in \mathbb{R} \mid \mathbb{P}_{\mu_i}(u_i = -w) > 0 \}$$

# Non-enforceable decisions

## Optimal self-enforcing rules

- ▶ The utility threshold (previously  $U_i^\theta$ ) is augmented.
- ▶ Countries below the threshold are overweighted and have veto power.
- ▶ In the illustration:  $U_1^{**} = 72/3^5 \approx 0.30 < U_1^\theta + 2/5 \approx 0.47 \Rightarrow 1$  has veto power.
- ▶ Countries above the threshold receive their efficient weight and do not have veto power.
- ▶ In the illustration:  $U_{2,3,4,5}^{**} = 84/3^5 \approx 0.35 > U_{2,3,4,5}^\theta + 1/5 \approx 0.33 \Rightarrow 2, 3, 4, 5$  have no veto power.
- ▶ In general, difficult to a priori identify countries with veto power. Possible under additional restrictions (e.g. in the apportionment model).

# Apportionment

## Model

Simpler model: countries only vary in their **populations,  $p_i$**

Under **sovereignty**:

- ▶ Each country now chooses independently which reforms to implement.
- ▶ In each country, for any given reform, a fraction  $q > 1/2$  of citizens agrees, gets utility +1, a fraction  $(1 - q)$  disagrees, gets utility -1.
- ▶  $U_i^\emptyset = (2q - 1)p_i$ .

Under **cooperation**:

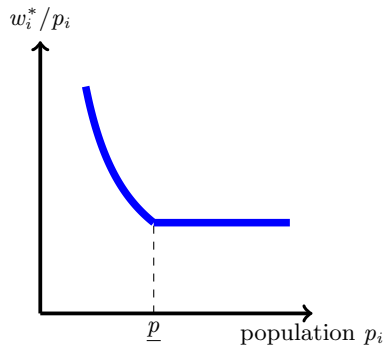
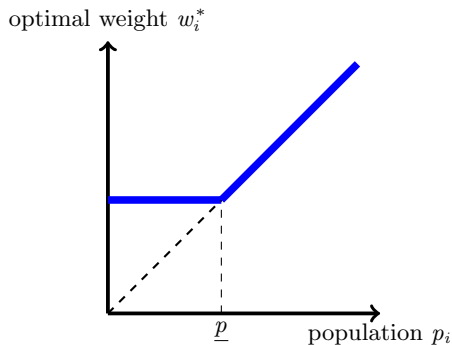
- ▶ Country  $i$ 's representative supports any given reform with probability  $1/2$ .
- ▶ A fraction  $q$  of citizens agrees with her representative.
- ▶ The reform yields utility  $e > 1$  for favorable citizens,  $-e$  for unfavorable ones.
- ▶  $e$ : per-capita efficiency gain from cooperation.
- ▶  $w_i^e = (2q - 1)ep_i/2$ .

# Apportionment

## Enforceable decisions

### Theorem

There exists  $\underline{p} \in \mathbb{R}$  such that any optimal voting rule is represented by  $[\mathbf{w}^*; 1/2]$ , with  $w_i^* = \max(p_i, \underline{p})$  for all  $i \in N$ .



# Apportionment

## Enforceable decisions

- ▶ Weights per capita decreases with the populations: degressive proportionality.
- ▶ New argument for DP that focuses on the bottom of the distribution, as opposed to existing models which recommend degressively proportional rules that have noticeable implications for medium to large states (Penrose square root law).
- ▶ Argument for a minimum representation threshold, as required explicitly in the Lisbon Treaty.
- ▶ Echoes the distribution of seats at the Electoral College which offers a compromise between Proportionality and Equality.



# Apportionment

## Non-Enforceable decisions

### Theorem

Any optimal self-enforcing rule is either the unanimity rule or a weighted majority rule for which no country has veto power. There exist a threshold  $\underline{e} > 0$ , and two non-increasing functions  $\delta^c, \delta^{eff} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ , such that  $\delta^c(e) \leq \delta^{eff}(e)$  for all  $e \in \mathbb{R}_+$ ,  $\lim_{e \rightarrow \infty} \delta^{eff}(e) < 1$ , and:

- (i) if  $\delta \geq \delta^{eff}(e)$ , any optimal self-enforcing rule is an efficient weighted majority rule,
- (ii) if  $\delta^c(e) \leq \delta < \delta^{eff}(e)$ , any optimal self-enforcing rule is a weighted majority rule, with overweighting of small countries,
- (iii) if  $\delta < \delta^c(e)$  and  $e \geq \underline{e}$ , the optimal self-enforcing rule is the unanimity rule,
- (iv) if  $\delta < \delta^c(e)$  and  $e < \underline{e}$ , there is no self-enforcing rule.

Moreover, for  $\delta^c(e) \leq \delta < \delta^{eff}(e)$ , there exists a minimal weight  $\underline{p}(e, \delta)$ , non-increasing in both  $e$  and  $\delta$ , such that any optimal self-enforcing rule is represented by  $[\mathbf{w}^{**}; 1/2]$ , defined by:

$$w_i^{**} = \max \left( p_i, \underline{p}(e, \delta) \right) \quad \text{for all } i \in N.$$

# Apportionment

## Non-Enforceable decisions

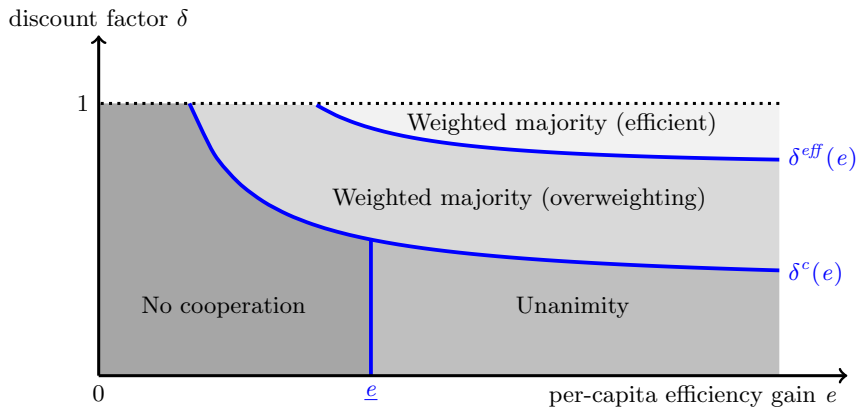


Figure: Optimal self-enforcing rule  $v^{**}$  in the model of apportionment

## Conclusion

Design of voting rules for international organizations with feasibility constraints:

- ▶ Countries' participation is voluntary.
- ▶ Collective decisions are generally not enforceable

We provided arguments for:

- ▶ Overweighting countries that have the lowest endogenous incentive to cooperate.
- ⇒ in the apportionment model: minimal representation for small countries.
- ▶ Granting a veto to some countries when decisions are not enforceable.

## Extension: bargaining

Alternative approach: Nash bargaining, instead of second-best:

$$\max_v \prod_{i \in N} (U_i(v) - U_i^\emptyset)$$

One can obtain a similar result:

- ▶ the bargained rule is weighted
- ▶ countries that gain relatively less from cooperation are relatively more overweighted:

$$U_i(v^*) - U_i^\emptyset < U_j(v^*) - U_j^\emptyset \quad \Rightarrow \quad w_i^* / w_i^e > w_j^* / w_j^e$$

Note: computing the bargained rule is more complex (non-linear optimization).